LF LearnFrenzy

QUANTITATIVE APTITUDE · FREE STUDY MATERIAL

Surds and Indices

Formulas & Shortcut Tricks

Every important formula, shortcut and exam trick for Surds and Indices in one place – with solved examples, common mistakes and a one-page revision sheet.

WHAT'S INSIDE

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USEFUL FOR

- SSC CGL / CHSL / MTS
- Bank PO & Clerk (IBPS, SBI, RBI)
- Railways (RRB NTPC, Group D)
- Campus placements (TCS, Infosys, Wipro, Accenture)
- UPSC CSAT & state PSC exams

Practise Surds and Indices free on LearnFrenzy – questions with instant explanations →

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1 Topic introduction

What it is, why it matters and how it is asked

Indices (exponents or powers) tell how many times a number is multiplied by itself: in a^n, a is the base and n the index. A small set of laws lets you multiply, divide and raise powers quickly, including zero, negative and fractional indices.

A surd is a root that cannot be simplified to a whole number, such as $\sqrt{2}$ or $\sqrt[3]{5}$. Exams ask you to simplify and add surds, rationalise denominators, find square roots of expressions like $7 + 4\sqrt{3}$, compare roots of different orders and solve exponential equations.

Why it matters in exams

- SSC papers ask simplification with powers and surds every time.
- Bank and placement tests use exponential equations.
- These laws also power Simplification, Logarithms and Approximation.

Where it is used

- Scientific notation and very large or small numbers.
- Compound growth and decay.
- Geometry – diagonals and heights with $\sqrt{2}$ and $\sqrt{3}$.

Common question patterns

Laws of indices
Product, quotient and power rules.

Zero, negative, fractional
a^0, a^{-n} and roots as powers.

Exponential equations
Same base; quadratics in 2^x or 3^x.

Simplifying surds
Take out square factors; add like surds.

Rationalising
Multiply by the conjugate.

Square root of $a + 2\sqrt{b}$
Write as $\sqrt{x} + \sqrt{y}$.

Comparing roots
Use a common index.

$x + 1/x$ and radicals
Conjugate pairs; infinite radicals.

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2 Important formulas

Cards marked MOST USED are the ones to revise first

Laws of indices

Product and quotient

MOST USED

$a^m \times a^n = a^{m+n}$; $a^m \div a^n = a^{m-n}$

Same base → add or subtract the indices.

EXAMPLE $2^5 \times 2^3 \div 2^4 = 2^4 = \mathbf{16}$.

Power of a power

MOST USED

$(a^m)^n = a^{mn}$; $(ab)^n = a^n b^n$; $(a/b)^n = a^n/b^n$

Multiply the indices.

EXAMPLE $(3^2)^3 = 3^6 = \mathbf{729}$.

Zero and negative indices

$a^0 = 1\ (a \neq 0)$; $a^{-n} = 1/a^n$

A negative index means the reciprocal.

EXAMPLE $5^{-2} = \mathbf{1/25} = 0.04$.

Fractional indices

MOST USED

$a^{1/n} = \sqrt[n]{a}$; $a^{m/n} = (\sqrt[n]{a})^m$

Take the root first – the numbers stay small.

EXAMPLE $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = \mathbf{9}$.

Exponential equations

Same base

MOST USED

$a^x = a^y \rightarrow x = y\ (a > 0, a \neq 1)$

Write both sides as powers of the same base.

EXAMPLE $4^{x+1} = 64 = 4^3 \rightarrow x + 1 = 3 \rightarrow x = \mathbf{2}$.

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Quadratic in a^x

Put $t = a^x$; solve the quadratic in t; then $a^x = t$

$4^x = (2^x)^2$, $9^x = (3^x)^2$.

EXAMPLE $4^x - 6 \cdot 2^x + 8 = 0 \rightarrow t = 2, 4 \rightarrow$ **x = 1 or 2**.

Surds

Simplifying and adding

MOST USED

$\sqrt{(a^2b)} = a\sqrt{b}$; $p\sqrt{b} + q\sqrt{b} = (p + q)\sqrt{b}$; $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$

Only like surds (same number under the root) can be added.

EXAMPLE $\sqrt{50} + \sqrt{18} = 5\sqrt{2} + 3\sqrt{2} =$ **$8\sqrt{2}$**.

Rationalising

MOST USED

$1/(\sqrt{a} + \sqrt{b}) = (\sqrt{a} - \sqrt{b})/(a - b)$ – multiply by the conjugate

$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$ removes the roots.

EXAMPLE $1/(\sqrt{5} + \sqrt{3}) =$ **$(\sqrt{5} - \sqrt{3})/2$**.

Square root of $a \pm 2\sqrt{b}$

$\sqrt{(a + 2\sqrt{b})} = \sqrt{x} + \sqrt{y}$ where $x + y = a$ and $xy = b$

Find two numbers with sum a and product b.

EXAMPLE $\sqrt{(7 + 4\sqrt{3})} = \sqrt{(7 + 2\sqrt{12})} \rightarrow 4 + 3, 4 \times 3 \rightarrow$ **$2 + \sqrt{3}$**.

Comparing roots

Raise every root to the LCM of the orders and compare the whole numbers

A higher-order root is not automatically smaller.

EXAMPLE $\sqrt{2}$ vs $\sqrt[3]{3} \rightarrow$ to the 6th power: 8 vs 9 → **$\sqrt[3]{3}$** is larger.

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3 Shortcut tricks & smart techniques

Each trick says when to use it – that matters as much as the trick

Same base first

MOST USED

USE WHEN Numbers like 8, 16, 27, 81, 125 appear in powers.

Base	Powers to know
2	4, 8, 16, 32, 64, 128, 256, 512, 1,024
3	9, 27, 81, 243, 729
5	25, 125, 625, 3,125
7	49, 343, 2,401

EXAMPLE $8^{2/3} \times 16^{1/4} = 2^2 \times 2 = \mathbf{8}$.

x + 1/x with conjugates

MOST USED

USE WHEN x is a surd like $3 + 2\sqrt{2}$ or $\sqrt{3} + \sqrt{2}$.

1. If $(p + \sqrt{q})(p - \sqrt{q}) = 1$, then 1/x is the conjugate.
2. $x + 1/x$ is then a whole number; $x^2 + 1/x^2 = (x + 1/x)^2 - 2$.

EXAMPLE $x = 3 + 2\sqrt{2} \rightarrow 1/x = 3 - 2\sqrt{2} \rightarrow x + 1/x = 6 \rightarrow x^2 + 1/x^2 = \mathbf{34}$.

Infinite radicals

USE WHEN A root that repeats for ever.

1. $\sqrt{(a + \sqrt{(a + \ldots)})} = x \rightarrow x^2 = a + x$; take the positive root.
2. $\sqrt{(a\sqrt{(a\sqrt{\ldots})})} = a$.

EXAMPLE $\sqrt{(12 + \sqrt{(12 + \ldots)})} \rightarrow x^2 - x - 12 = 0 \rightarrow x = \mathbf{4}$.

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4 Solved examples

The full method on the left, the exam shortcut on the right

1 LAWS

Simplify $(2^{-3} \times 2^{5}) \div 2^{-1}$.

STEP BY STEP

1. Numerator: $2^{-3+5} = 2^{2}$.
2. Divide: $2^{2-(-1)} = 2^{3}$.
3. $= 8$.

SHORTCUT

Indices: $-3 + 5 + 1 = 3$.

Why it works: Trap: 2 – subtracting –1 as if it were +1.

Answer: 8

2 FRACTIONAL

Find the value of $81^{3/4} \div 9^{1/2}$.

STEP BY STEP

1. $81^{3/4} = (\sqrt[4]{81})^{3} = 3^{3} = 27$.
2. $9^{1/2} = 3$.
3. $27 \div 3 = 9$.

SHORTCUT

$3^{3} \div 3^{1} = 3^{2}$.

Why it works: Write both as powers of 3.

Answer: 9

3 EQUATION

Solve $3^{2x-1} = 243$.

STEP BY STEP

1. $243 = 3^{5}$.
2. $2x - 1 = 5$.
3. $x = 3$.

SHORTCUT

Match the indices.

Why it works: Same base → equal indices.

Answer: x = 3

4 COMMON FACTOR

Solve $2^{x+3} - 2^{x} = 112$.

STEP BY STEP

1. $2^{x}(2^{3} - 1) = 112$.
2. $7 \times 2^{x} = 112 \rightarrow 2^{x} = 16$.
3. $x = 4$.

SHORTCUT

Take out the smallest power.

Why it works: Trap: treating $2^{x+3} - 2^{x}$ as 2^{3}.

Answer: x = 4

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5 QUADRATIC

Solve $9^x - 12 \cdot 3^x + 27 = 0$.

STEP BY STEP

1. Let $t = 3^x$: $t^2 - 12t + 27 = 0$.
2. $(t - 3)(t - 9) = 0 \rightarrow t = 3$ or 9.
3. $x = 1$ or 2.

SHORTCUT

Roots with sum 12 and product 27.

Why it works: $9^x = (3^x)^2$.

Answer: x = 1 or 2

6 RELATION

If $2^a = 3^b = 6^c$, express c in terms of a and b.

STEP BY STEP

1. Let each = k: $2 = k^{1/a}$, $3 = k^{1/b}$, $6 = k^{1/c}$.
2. $6 = 2 \times 3 \rightarrow 1/c = 1/a + 1/b$.
3. $c = ab/(a + b)$.

SHORTCUT

Multiply the bases, add the reciprocals.

Why it works: Trap: $c = a + b$ – adding the indices directly.

Answer: c = ab/(a + b)

7 SURDS

Simplify $\sqrt{98} - \sqrt{50} + \sqrt{32}$.

STEP BY STEP

1. $\sqrt{98} = 7\sqrt{2}$; $\sqrt{50} = 5\sqrt{2}$; $\sqrt{32} = 4\sqrt{2}$.
2. $7\sqrt{2} - 5\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$.

SHORTCUT

Take out the largest square factor.

Why it works: Trap: $\sqrt{80}$ – adding numbers under the roots.

Answer: $6\sqrt{2}$

8 RATIONALISE

Rationalise the denominator of $6/(\sqrt{7} - 2)$.

STEP BY STEP

1. Multiply by $(\sqrt{7} + 2)/(\sqrt{7} + 2)$.
2. Denominator: $7 - 4 = 3$.
3. $6(\sqrt{7} + 2)/3 = 2\sqrt{7} + 4$.

SHORTCUT

Conjugate changes the sign in the middle.

Why it works: Trap: $(\sqrt{7} - 2)$ on top – forgetting the conjugate's sign.

Answer: $2\sqrt{7} + 4$

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9 SQUARE ROOT

Find $\sqrt{(11 - 2\sqrt{30})}$.

STEP BY STEP

1. Need $x + y = 11$, $xy = 30 \rightarrow x = 6, y = 5$.
2. $\sqrt{(11 - 2\sqrt{30})} = \sqrt{6} - \sqrt{5}$.

SHORTCUT

Larger number first so the result is positive.

Why it works: Trap: $\sqrt{5} - \sqrt{6}$ – a negative number.

Answer: $\sqrt{6} - \sqrt{5}$

10 COMPARING

Arrange $\sqrt[3]{4}$, $\sqrt[4]{5}$ and $\sqrt{3}$ in order and name the largest and smallest.

STEP BY STEP

1. LCM of 3, 4, 2 = 12.
2. $\sqrt[3]{4} \rightarrow 4^4 = 256$; $\sqrt[4]{5} \rightarrow 5^3 = 125$; $\sqrt{3} \rightarrow 3^6 = 729$.
3. Order: $\sqrt[4]{5} < \sqrt[3]{4} < \sqrt{3}$.

SHORTCUT

Compare the 12th powers.

Why it works: Same index makes the comparison fair.

Answer: Largest $\sqrt{3}$, smallest $\sqrt[4]{5}$

11 X + 1/X

If $x = \sqrt{3} + \sqrt{2}$, find $x^2 + 1/x^2$.

STEP BY STEP

1. $1/x = \sqrt{3} - \sqrt{2}$ (since $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = 1$).
2. $x + 1/x = 2\sqrt{3} \rightarrow (x + 1/x)^2 = 12$.
3. $x^2 + 1/x^2 = 12 - 2 = 10$.

SHORTCUT

$x^2 = 5 + 2\sqrt{6}$, $1/x^2 = 5 - 2\sqrt{6} \rightarrow 10$.

Why it works: The surd parts cancel.

Answer: 10

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5 Important exam tips

Patterns, traps and habits that save marks

Frequently asked question patterns

- Simplification with laws of indices.
- Fractional and negative powers.
- Exponential equations; quadratics in 2^x or 3^x.
- Adding and simplifying surds.
- Rationalising with conjugates.
- Square root of $a \pm 2\sqrt{b}$.
- Comparing roots; $x + 1/x$ values.

Common mistakes to avoid

- Multiplying the base by the index ($2^3 = 6$).
- Adding unlike surds ($\sqrt{2} + \sqrt{3} = \sqrt{5}$).
- Forgetting the conjugate's sign.
- Taking a negative square root.
- Comparing roots of different orders directly.
- Writing $a^0 = 0$.

Points to remember

- $a^m a^n = a^{m+n}$.
- $(a^m)^n = a^{mn}$.
- $a^{-n} = 1/a^n$.
- $a^{m/n} = (\sqrt[n]{a})^m$.
- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$.

Speed and accuracy

- Convert to the smallest base first.
- Take the root before the power.
- Learn powers of 2 to 2^{10} and of 3 to 3^6.
- Use the 12th power to compare $\sqrt[3]{}$, $\sqrt[4]{}$ and $\sqrt{}$.

Preparation strategy

- Learn the laws by heart – they are used in many topics.
- Practise on LearnFrenzy in order: Easy → Medium → Hard → Exam Pattern Mix → Data Sufficiency.
- Aim for 30–45 seconds per question.
- Then revise Logarithms – the same laws in reverse.
- Read the Quick Revision Sheet (next page) the night before the exam.

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6 Quick revision sheet

Everything on one page for the night before the exam

Formulas

Product	$a^m \cdot a^n = a^{m+n}$	Quotient	$a^m \div a^n = a^{m-n}$
Power	$(a^m)^n = a^{mn}$	Zero	$a^0 = 1$
Negative	$a^{-n} = 1/a^n$	Fractional	$a^{m/n} = (\sqrt[n]{a})^m$
Same base	$a^x = a^y \rightarrow x = y$	Surd product	$\sqrt{a} \cdot \sqrt{b} = \sqrt{(ab)}$
Rationalise	$1/(\sqrt{a} + \sqrt{b}) = (\sqrt{a} - \sqrt{b})/(a - b)$	$\sqrt{(a + 2\sqrt{b})}$	$\sqrt{x} + \sqrt{y}, x + y = a, xy = b$
$x^2 + 1/x^2$	$(x + 1/x)^2 - 2$	Infinite radical	$\sqrt{(a + \sqrt{(a + \ldots)})} = (1 + \sqrt{(1 + 4a)})/2$

Shortcut tricks

- Same base first.
- Root before power.
- Like surds only add.
- Conjugate → 1/x.
- Compare with a common index.
- Take the positive root.

Must-know numbers

Powers of 2	2^6 64 · 2^7 128 · 2^8 256 · 2^9 512 · 2^{10} 1,024
Powers of 3 and 5	3^4 81 · 3^5 243 · 3^6 729 · 5^3 125 · 5^4 625
Roots	$\sqrt{2} = 1.414$ · $\sqrt{3} = 1.732$ · $\sqrt{5} = 2.236$ · $\sqrt[3]{2} = 1.26$

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7 A message from LearnFrenzy

For everyone preparing for placements, competitive and government exams

Keep going – you are closer than you think

Success in examinations is not just about how many hours you study. It is about how effectively you understand concepts, practise questions, and improve yourself every day.

Every formula you learn, every problem you solve, and every mistake you correct brings you one step closer to your goal.

Whether you are preparing for campus placements, competitive examinations, or government jobs, remember that consistency, confidence, and smart preparation make a real difference.

Don’t just memorize formulas. Understand the concepts, practise smart techniques, and develop the confidence to solve problems independently.

Your hard work today can create the opportunities you dream of tomorrow.

— Team LearnFrenzy

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