LF LearnFrenzy

QUANTITATIVE APTITUDE · FREE STUDY MATERIAL

Heights and Distances

Formulas & Shortcut Tricks

Every important formula, shortcut and exam trick for Heights and Distances in one place – with solved examples, common mistakes and a one-page revision sheet.

WHAT'S INSIDE

1. Topic introduction
2. Important formulas
3. Shortcut tricks & smart techniques
4. Solved examples
5. Important exam tips
6. Quick revision sheet
7. A message from LearnFrenzy

USEFUL FOR

- SSC CGL / CHSL / CPO
- Railways (RRB NTPC, Group D)
- Bank PO & Clerk (IBPS, SBI, RBI)
- Campus placements (TCS, Infosys, Wipro, Accenture)
- UPSC CSAT & state PSC exams

Practise Heights and Distances free on LearnFrenzy – questions with instant explanations →

© 2026 LearnFrenzy · Free for personal study

1 Topic introduction

What it is, why it matters and how it is asked

Heights and Distances uses right-angled triangles to find heights and distances that cannot be measured directly – the height of a tower, the width of a river or the distance of a ship from a lighthouse.

The angle of elevation is measured upward from the horizontal to an object above the eye; the angle of depression is measured downward to an object below. Almost every question uses 30°, 45° or 60° and only one ratio: tan = height ÷ base distance.

Why it matters in exams

- SSC papers ask one or two questions every time.
- Railway and state exams use towers, ladders and shadows.
- Most questions need just one standard angle.

Where it is used

- Surveying land and buildings.
- Navigation – ships, lighthouses and aircraft.
- Architecture, ramps and ladders.

Common question patterns

One angle
Tower, shadow, ladder or kite.

Elevation and depression
From the top of a cliff or lighthouse.

Walking towards a tower
Angle changes from α to β.

Opposite sides
Two observers on either side.

Flagstaff on a building
Two angles to the bottom and top.

Observer's height
Add the eye height to the answer.

Broken trees
The top touches the ground.

Speed and complementary angles
Moving cars; $h^2 = ab$.

© 2026 LearnFrenzy · Free for personal study

2 Important formulas

Cards marked MOST USED are the ones to revise first

Trigonometric ratios

θ	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	not defined

The three ratios

MOST USED

sin θ = perpendicular/hypotenuse; cos θ = base/hypotenuse; tan θ = perpendicular/base

Height and ground distance → tan. Slant length → sin or cos.

EXAMPLE Shadow 20 m when the sun is at 45° → height = 20 × tan 45° = **20 m**.

Elevation and depression

MOST USED

Angle of depression from A to B = angle of elevation from B to A

They are alternate angles between parallel horizontals.

EXAMPLE From the top of a 30 m cliff, depression of a boat 30° → distance = 30/tan 30° = **30√3 m**.

Standard set-ups

One angle

MOST USED

h = d × tan θ; d = h × cot θ

h is the height, d the horizontal distance.

EXAMPLE d = 50 m, θ = 60° → h = **50√3 m**.

Two angles, same side

MOST USED

h = d/(cot α − cot β) (d = distance between the two points, α < β)

The observer walks d towards the tower.

EXAMPLE 30° and 60°, d = 40 m → h = 40/(√3 − 1/√3) = **20√3 m**.

© 2026 LearnFrenzy · Free for personal study

Two angles, opposite sides

> **Distance between the observers = h (cot α + cot β)**

Add the two ground distances.

EXAMPLE h = 30 m, 30° and 60° → $30(\sqrt{3} + 1/\sqrt{3}) = \mathbf{40\sqrt{3}\ m}$.

Complementary angles

> **Angles θ and 90° − θ from distances a and b → $h^2 = ab$**

$\tan\theta \times \tan(90° - \theta) = 1$.

EXAMPLE a = 4 m, b = 9 m → $h = \sqrt{36} = \mathbf{6\ m}$.

Ladders and broken trees

Ladder against a wall

> **Height reached = L sin θ; foot from wall = L cos θ**

The ladder is the hypotenuse.

EXAMPLE Reaches 6 m up at 60° → $L = 6 / \sin 60° = \mathbf{4\sqrt{3}\ m}$.

Broken tree

> **Top touches the ground x from the foot at θ: standing part = x tan θ; broken part = x / cos θ**

Original height = standing part + broken part.

EXAMPLE x = 10 m, θ = 30° → $10/\sqrt{3} + 20/\sqrt{3} = \mathbf{10\sqrt{3}\ m}$.

© 2026 LearnFrenzy · Free for personal study

3 Shortcut tricks & smart techniques

Each trick says when to use it – that matters as much as the trick

Special triangles

MOST USED

USE WHEN Any 30°, 45° or 60° triangle.

Triangle	Sides in the ratio
30° – 60° – 90°	$1 : \sqrt{3} : 2$ (opposite 30°, 60°, 90°)
45° – 45° – 90°	$1 : 1 : \sqrt{2}$

EXAMPLE Hypotenuse 20 at 30° → opposite 10, adjacent $\mathbf{10\sqrt{3}}$.

Walking from 30° to 60°

MOST USED

USE WHEN The angle changes from 30° to 60° as the observer walks d.

1. Height = $d\sqrt{3}/2$.
2. Remaining distance to the tower = $d/2$.

EXAMPLE d = 40 m → height $20\sqrt{3}$ m, still **20 m** to go.

45° means equal

USE WHEN An angle of 45° appears.

1. Height = horizontal distance.
2. Write h for both and save a step.

EXAMPLE Kite string at 45°, kite 35 m up → 35 m along the ground.

Surds to decimals

USE WHEN Options are in decimals.

1. $\sqrt{3} = 1.732$, $\sqrt{2} = 1.414$, $1/\sqrt{3} = 0.577$.
2. Multiply only at the last step.

EXAMPLE $20\sqrt{3} = 20 \times 1.732 = \mathbf{34.64}$.

© 2026 LearnFrenzy · Free for personal study

4 Solved examples

The full method on the left, the exam shortcut on the right

1 SHADOW

A tower casts a shadow $25\sqrt{3}$ m long when the sun's altitude is 30°. Find the height of the tower.

STEP BY STEP

1. $\tan 30° = h/25\sqrt{3}$.
2. $h = 25\sqrt{3} \times 1/\sqrt{3}$.
3. = 25 m.

SHORTCUT

30° → height = shadow/$\sqrt{3}$.

Why it works: Trap: 75 m – using tan 60° instead of tan 30°.

Answer: 25 m

2 LADDER

A 20 m ladder leans against a wall making 60° with the ground. How high up the wall does it reach?

STEP BY STEP

1. $\sin 60° = h/20$.
2. $h = 20 \times \sqrt{3}/2$.
3. = $10\sqrt{3}$ m.

SHORTCUT

1 : $\sqrt{3}$: 2 → 10, $10\sqrt{3}$, 20.

Why it works: The ladder is the hypotenuse – use sin.

Answer: $10\sqrt{3}$ m

3 DEPRESSION

From the top of a 75 m lighthouse, the angle of depression of a ship is 30°. How far is the ship from the lighthouse?

STEP BY STEP

1. Angle of elevation from the ship = 30°.
2. $\tan 30° = 75/d$.
3. $d = 75\sqrt{3}$ m.

SHORTCUT

$d = h \times \cot 30° = h\sqrt{3}$.

Why it works: Depression from the top = elevation from the ship.

Answer: $75\sqrt{3}$ m

© 2026 LearnFrenzy · Free for personal study

4 WALKING TOWARDS

The angle of elevation of the top of a tower changes from 30° to 60° as a man walks 50 m towards it. Find the height of the tower.

STEP BY STEP

1. Let the nearer distance be x: $h = x\sqrt{3}$.
2. Also $h = (x + 50)/\sqrt{3}$.
3. $3x = x + 50 \rightarrow x = 25$.
4. $h = 25\sqrt{3}$ m.

SHORTCUT

30° → 60°: $h = d\sqrt{3}/2$.

Why it works: Write h twice and equate.

Answer: $25\sqrt{3}$ m

5 OPPOSITE SIDES

Two men on opposite sides of a 60 m tower see its top at angles of 30° and 45°. Find the distance between them ($\sqrt{3} = 1.732$).

STEP BY STEP

1. Distances: $60\sqrt{3}$ and 60.
2. Total = $60(\sqrt{3} + 1)$.
3. = 60 × 2.732 = 163.92 m.

SHORTCUT

h (cot 30° + cot 45°).

Why it works: Trap: subtracting – they are on opposite sides.

Answer: 163.92 m

6 FLAGSTAFF

From a point 40 m from a building, the angles of elevation of the bottom and top of a flagstaff on it are 30° and 45°. Find the length of the flagstaff ($\sqrt{3} = 1.732$).

STEP BY STEP

1. Top: 40 × tan 45° = 40 m.
2. Building: 40 × tan 30° = $40/\sqrt{3} \approx 23.09$ m.
3. Flagstaff = 40 – 23.09 = 16.91 m.

SHORTCUT

$40(1 - 1/\sqrt{3})$.

Why it works: The flagstaff is the difference of two heights.

Answer: 16.91 m

7 OBSERVER'S HEIGHT

A girl 1.6 m tall stands 24 m from a pole and sees its top at 45°. Find the height of the pole.

STEP BY STEP

1. Height above her eyes = 24 × tan 45° = 24 m.
2. Pole = 24 + 1.6 = 25.6 m.

SHORTCUT

45° → equal, then add eye height.

Why it works: Trap: 24 m – the observer's height forgotten.

Answer: 25.6 m

© 2026 LearnFrenzy · Free for personal study

8 BROKEN TREE

A tree breaks in a storm and its top touches the ground 12 m from the foot, making 30° with the ground. Find the original height.

STEP BY STEP

1. Standing part = 12 tan 30° = $4\sqrt{3}$.
2. Broken part = $12/\cos 30° = 8\sqrt{3}$.
3. Height = $12\sqrt{3}$ m.

SHORTCUT

$x \tan\theta + x \sec\theta = 12(1/\sqrt{3} + 2/\sqrt{3})$.

Why it works: Trap: $4\sqrt{3}$ – giving only the standing part.

Answer: $12\sqrt{3}$ m

9 COMPLEMENTARY

The angles of elevation of the top of a tower from two points 8 m and 18 m from its base, on the same line, are complementary. Find its height.

STEP BY STEP

1. $\tan\theta = h/8$ and $\tan(90° - \theta) = h/18$.
2. Product = 1 → $h^2 = 144$.
3. h = 12 m.

SHORTCUT

$h = \sqrt{(8 \times 18)}$.

Why it works: $\tan\theta \cdot \cot\theta = 1$.

Answer: 12 m

10 SPEED

From the top of a $50\sqrt{3}$ m tower, the angle of depression of a car changes from 30° to 60° in 10 seconds as it approaches. Find its speed in km/h.

STEP BY STEP

1. Distance at 30° = $50\sqrt{3} \times \sqrt{3} = 150$ m.
2. At 60° = $50\sqrt{3}/\sqrt{3} = 50$ m.
3. Covered 100 m in 10 s = 10 m/s = 36 km/h.

SHORTCUT

30° → 60°: distance covered = $2h/\sqrt{3}$.

Why it works: Speed = distance ÷ time; × 18/5 for km/h.

Answer: 36 km/h

11 TWO SHIPS

From the top of a 60 m lighthouse, the angles of depression of two ships on the same side are 30° and 45°. Find the distance between the ships ($\sqrt{3} = 1.732$).

STEP BY STEP

1. Far ship: $60\sqrt{3}$; near ship: 60.
2. Gap = $60(\sqrt{3} - 1)$.
3. = 60 × 0.732 = 43.92 m.

SHORTCUT

$h(\cot 30° - \cot 45°)$.

Why it works: Same side → subtract the distances.

Answer: 43.92 m

© 2026 LearnFrenzy · Free for personal study

5 Important exam tips

Patterns, traps and habits that save marks

Frequently asked question patterns

- Shadows, ladders and kites with one angle.
- Depression from a cliff or lighthouse.
- Observer walking towards a tower.
- Two observers or ships.
- Flagstaff or statue on a building.
- Broken trees and observer's height.
- Speed of a car; complementary angles.

Common mistakes to avoid

- Swapping tan 30° and tan 60°.
- Using sin where tan is needed.
- Forgetting the observer's height.
- Adding distances when both are on the same side.
- Giving only part of a broken tree.
- Measuring depression from the vertical.

Points to remember

- tan = height ÷ base.
- tan 30° = 1/$\sqrt{3}$, tan 60° = $\sqrt{3}$.
- Depression = elevation.
- 30° → 60°: h = d$\sqrt{3}$/2.
- Complementary: $h^2 = ab$.

Speed and accuracy

- Draw a quick sketch every time.
- Use 1 : $\sqrt{3}$: 2 instead of ratios.
- Keep surds until the last step.
- Check: higher angle → nearer point.

Preparation strategy

- Learn the value table for 30°, 45° and 60°.
- Practise on LearnFrenzy in order: Easy → Medium → Hard → Exam Pattern Mix → Data Sufficiency.
- Aim for 45–60 seconds per question.
- Keep an error log of tan/cot slips.
- Read the Quick Revision Sheet (next page) the night before the exam.

© 2026 LearnFrenzy · Free for personal study

6 Quick revision sheet

Everything on one page for the night before the exam

Formulas

tan θ	**height ÷ base**	sin θ	**height ÷ hypotenuse**
cos θ	**base ÷ hypotenuse**	One angle	**h = d tan θ**
Same side (α < β)	**h = d/(cot α − cot β)**	Opposite sides	**gap = h(cot α + cot β)**
Complementary	**h² = ab**	Ladder	**h = L sin θ**
Broken tree	**x tan θ + x/cos θ**	Observer	**answer + eye height**

Shortcut tricks

- 30-60-90 → 1 : √3 : 2.
- 45-45-90 → 1 : 1 : √2.
- 45° → height = distance.
- 30° → 60°: h = d√3/2.
- Same side → subtract.
- Opposite sides → add.

Must-know numbers

tan	tan 30° = 0.577 · tan 45° = 1 · tan 60° = 1.732
Roots	√2 = 1.414 · √3 = 1.732 · √3 − 1 = 0.732 · √3 + 1 = 2.732

© 2026 LearnFrenzy · Free for personal study

7 A message from LearnFrenzy

For everyone preparing for placements, competitive and government exams

Keep going – you are closer than you think

Success in examinations is not just about how many hours you study. It is about how effectively you understand concepts, practise questions, and improve yourself every day.

Every formula you learn, every problem you solve, and every mistake you correct brings you one step closer to your goal.

Whether you are preparing for campus placements, competitive examinations, or government jobs, remember that consistency, confidence, and smart preparation make a real difference.

Don’t just memorize formulas. Understand the concepts, practise smart techniques, and develop the confidence to solve problems independently.

Your hard work today can create the opportunities you dream of tomorrow.

— Team LearnFrenzy

Practise **Heights and Distances** free on LearnFrenzy – questions with instant explanations →

© 2026 LearnFrenzy · Free for personal study