LF LearnFrenzy

QUANTITATIVE APTITUDE · FREE STUDY MATERIAL

Logarithms

Formulas & Shortcut Tricks

Every important formula, shortcut and exam trick for Logarithms in one place – with solved examples, common mistakes and a one-page revision sheet.

WHAT'S INSIDE

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USEFUL FOR

- Campus placements (TCS, Infosys, Wipro, Accenture)
- Bank PO (IBPS, SBI)
- Railways (RRB NTPC)
- UPSC CSAT & state PSC exams
- SSC CGL

Practise Logarithms free on LearnFrenzy – questions with instant explanations →

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1 Topic introduction

What it is, why it matters and how it is asked

A logarithm answers the question “to what power must the base be raised to give this number?”. Since $2^6 = 64$, $\log_2 64 = 6$. Logarithms turn multiplication into addition and powers into multiplication, which is why they were invented to simplify huge calculations.

Exam questions test the three laws (product, quotient, power), change of base, the identity a^(log$_a$ x) = x, solving log equations with the domain in mind, and using log 2 and log 3 to count the digits of huge numbers like 2^{50}.

Why it matters in exams

- Placement tests (TCS, Infosys, AMCAT) ask 1–2 log questions regularly.
- Bank PO, CSAT and state exams use log laws, equations and digit counting.
- Logs reappear in compound growth, sound and earthquake scales and algorithms.

Where it is used

- Finding how many years compound growth takes.
- Scales such as decibels, pH and the Richter scale.
- Computer science: binary search and algorithm speed ($\log_2 n$).

Common question patterns

Definition
Find $\log_a x$, or x from a log value.

Laws
Simplify sums and differences of logs.

Change of base
Chains like $\log_2 3 \times \log_3 4 \times \ldots$.

Power identity
Simplify a^(log$_a$ x) and a^(log b).

Equations
Solve log equations and reject invalid roots.

Log tables
Use log 2 and log 3 to find log 18, log 72 …

Digit counting
Digits in 2^{50} or zeros after the point in $(1/2)^{20}$.

Telescoping
Sums of logs of fractions that cancel.

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2 Important formulas

Cards marked MOST USED are the ones to revise first

Definition and basic values

What a logarithm means

MOST USED

$$\log_a x = y \Leftrightarrow a^y = x \ (a > 0, a \neq 1, x > 0)$$

The log is the power. Logs of zero or negative numbers do not exist.

EXAMPLE $2^6 = 64$, so $\log_2 64 = 6$.

Special values

$$\log_a 1 = 0;\ \log_a a = 1;\ \log_a (1/a) = -1$$

Any base to the power 0 is 1; to the power 1 is itself.

EXAMPLE $\log_7 1 = 0$ and $\log_7 7 = 1$ → their sum is **1**.

The three laws

Product law

MOST USED

$$\log_a (mn) = \log_a m + \log_a n$$

Multiplication inside becomes addition outside.

EXAMPLE $\log 2 + \log 5 = \log 10 = 1$.

Quotient law

MOST USED

$$\log_a (m/n) = \log_a m - \log_a n$$

Division inside becomes subtraction outside.

EXAMPLE $\log_3 81 - \log_3 9 = \log_3 9 = 2$.

Power law

MOST USED

$$\log_a (m^n) = n \times \log_a m$$

The power comes down in front.

EXAMPLE $\log_2 (8^5) = 5 \times \log_2 8 = 15$.

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Changing the base

Change of base

MOST USED

$\log_a b = \log b / \log a$ (any common base)

Convert to base 10, or to a base both numbers are powers of.

EXAMPLE $\log_4 8 = \log 2^3 / \log 2^2 =$ **3/2**.

Reciprocal and chain rules

MOST USED

$\log_a b \times \log_b a = 1$; $\log_a b \times \log_b c = \log_a c$

In a chain, each middle base cancels the next number.

EXAMPLE $\log_2 5 \times \log_5 32 = \log_2 32 =$ **5**.

Powers in the base and the number

$\log_{a^m} b^n = (n/m) \times \log_a b$

Powers of the base go underneath, powers of the number on top.

EXAMPLE $\log_{\sqrt{2}} 8 = 3/(1/2) =$ **6**.

The power identity

$a^{\log_a x} = x$; $a^{\log b} = b^{\log a}$

Raising the base to its own log undoes the log.

EXAMPLE $5^{\log_5 7} =$ **7**.

Equations and big numbers

Solving log equations

MOST USED

Combine the logs, remove them by writing the power form, then check every root in the original logs

A root that makes any log's argument zero or negative must be rejected.

EXAMPLE $\log_2(x+3) + \log_2(x-3) = 4 \rightarrow x^2 - 9 = 16 \rightarrow x = \pm 5 \rightarrow$ only **x = 5** is valid.

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Number of digits

MOST USED

> **Digits in N = (integer part of $\log_{10} N$) + 1**

The characteristic (integer part) tells the size; the mantissa tells the digits.

EXAMPLE 2^{50}: 50 × 0.3010 = 15.05 → **16 digits**.

Zeros after the decimal point

> **If $\log_{10} N = -k.\text{something}$ ($N < 1$), the number of zeros after the point = k**

Write the log as a negative characteristic plus a positive mantissa.

EXAMPLE $(1/2)^{20}$: log = −20 × 0.3010 = −6.02 → **6 zeros** after the point.

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3 Shortcut tricks & smart techniques

Each trick says when to use it – that matters as much as the trick

Standard log values

MOST USED

USE WHEN Questions that give log 2 and log 3, or ask for logs of numbers built from 2, 3, 5 and 7.

log 2	log 3	log 5	log 7
0.3010	0.4771	0.6990 (= 1 – log 2)	0.8451

EXAMPLE log 72 = 3 log 2 + 2 log 3 = 0.9030 + 0.9542 = **1.8572**.

Write everything with one base

MOST USED

USE WHEN Bases and numbers are powers of the same prime (2, 4, 8, 16 or 3, 9, 27 …).

1. Rewrite each number as a power of the prime.
2. Use $\log_{a^m} b^n = (n/m) \log_a b$.

EXAMPLE $\log_9 27 = \log_{3^2} 3^3 = \mathbf{3/2}$.

Chains cancel

USE WHEN Products like $\log_2 3 \times \log_3 4 \times \log_4 5 \times \dots$.

1. Each log's number is the next log's base.
2. Only the first base and the last number remain.

EXAMPLE $\log_2 3 \times \log_3 4 \times \dots \times \log_{15} 16 = \log_2 16 = \mathbf{4}$.

Telescoping sums

USE WHEN Sums of logs of (k + 1)/k.

1. Combine into one log of a product.
2. The fractions cancel to last ÷ first.

EXAMPLE log(2/1) + log(3/2) + … + log(100/99) = log 100 = **2**.

Check the domain last

USE WHEN Every log equation.

1. After solving, substitute each root into every original log.
2. Reject roots that make any argument ≤ 0 or the base ≤ 0 or 1.

EXAMPLE log(x + 2) + log(x – 1) = 1 gives x = 3 or –4; x = –4 makes x – 1 negative → only **x = 3**.

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4 Solved examples

The full method on the left, the exam shortcut on the right

1 DEFINITION

Find $\log_3 (1/81)$.

STEP BY STEP

1. $1/81 = 3^{-4}$.
2. $\log_3 3^{-4} = -4$.

SHORTCUT

$81 = 3^4$, and 1/81 flips the sign → −4.

Why it works: A reciprocal turns the power negative.

Answer: −4

2 PRODUCT LAW

Simplify log 25 + log 40.

STEP BY STEP

1. log 25 + log 40 = log (25 × 40).
2. = log 1,000 = 3.

SHORTCUT

25 × 40 = 1,000 → 3.

Why it works: Adding logs multiplies the numbers.

Answer: 3

3 CHANGE OF BASE

Find $\log_8 32$.

STEP BY STEP

1. $8 = 2^3$ and $32 = 2^5$.
2. $\log_8 32 = 5/3$.

SHORTCUT

$\log_{2^3} 2^5 = 5/3$.

Why it works: Both numbers are powers of 2.

Answer: 5/3

4 CHAIN

Find the value of $\log_2 3 \times \log_3 4 \times \log_4 5 \times \ldots \times \log_{15} 16$.

STEP BY STEP

1. Write each as log(k + 1)/log k.
2. All middle terms cancel: log 16 / log 2.
3. $= \log_2 16 = 4$.

SHORTCUT

First base 2, last number 16 → $\log_2 16 = 4$.

Why it works: Each number is the next base, so they cancel in pairs.

Answer: 4

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5 LOG TABLE

Given log 2 = 0.3010 and log 3 = 0.4771, find log 1.8.

STEP BY STEP

1. 1.8 = 18/10 = (2 × 3^2)/10.
2. log 1.8 = log 2 + 2 log 3 − 1.
3. = 0.3010 + 0.9542 − 1 = 0.2552.

SHORTCUT

log 18 = 1.2552 → subtract 1.

Why it works: Every factor of 18 has a known log.

Answer: 0.2552

6 UNKNOWN BASE

If $\log_x 343 = 3$, find x.

STEP BY STEP

1. $x^3 = 343$.
2. x = 7.

SHORTCUT

$\sqrt[3]{343} = 7$.

Why it works: Write the log in power form.

Answer: 7

7 EQUATION

Solve log(x + 2) + log(x − 1) = 1 (logs to base 10).

STEP BY STEP

1. log[(x + 2)(x − 1)] = 1 → (x + 2)(x − 1) = 10.
2. $x^2 + x - 12 = 0$ → x = 3 or x = −4.
3. x = −4 makes log(x − 1) undefined → x = 3.

SHORTCUT

Try options: x = 3 → log 5 + log 2 = 1 ✓.

Why it works: Logs exist only for positive numbers, so check each root.

Answer: x = 3

8 DIGITS

How many digits does 3^{40} have? (log 3 = 0.4771)

STEP BY STEP

1. log 3^{40} = 40 × 0.4771 = 19.084.
2. Characteristic 19 → 19 + 1 = 20 digits.

SHORTCUT

Integer part + 1.

Why it works: A number between 10^{19} and 10^{20} has 20 digits.

Answer: 20

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9 ZEROS AFTER THE POINT

How many zeros are there between the decimal point and the first significant digit of $(1/3)^{10}$? (log 3 = 0.4771)

STEP BY STEP

1. log $(1/3)^{10}$ = −4.771 = −5 + 0.229.
2. Characteristic −5 → 4 zeros after the point.

SHORTCUT

Integer part of 4.771 is 4 → 4 zeros.

Why it works: The value is about 0.0000169.

Answer: 4

10 POWER IDENTITY

Find $7^{\log_7 12} + 2^{\log_4 81}$.

STEP BY STEP

1. $7^{\log_7 12}$ = 12.
2. $\log_4 81 = \log_2 9 \rightarrow 2^{\log_2 9} = 9$.
3. Total = 21.

SHORTCUT

Each term undoes its log: 12 + 9.

Why it works: $\log_4 81 = \log_{2^2} 9^2 = \log_2 9$.

Answer: 21

11 TELESCOPING

Find log(2/1) + log(3/2) + log(4/3) + … + log(100/99).

STEP BY STEP

1. Combine: log(2/1 × 3/2 × … × 100/99).
2. Everything cancels except 100/1.
3. log 100 = 2.

SHORTCUT

Last numerator ÷ first denominator = 100 → 2.

Why it works: Adding logs multiplies the fractions, which telescope.

Answer: 2

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5 Important exam tips

Patterns, traps and habits that save marks

Frequently asked question patterns

- Values like $\log_2 64$, $\log_3 (1/81)$, $\log_8 32$.
- Sums and differences of logs that make a round number.
- Change-of-base chains.
- a^(log$_a$ x) and a^(log b) = b^(log a).
- Log equations with a root to reject.
- Using log 2 and log 3 for log 18, log 72, log 1.8.
- Digits in 2^{50} or 3^{40}; zeros after the point.

Common mistakes to avoid

- Writing log(m + n) = log m + log n – there is no such law.
- Writing log m × log n = log(mn).
- Forgetting to reject roots that make an argument negative or zero.
- Using log 5 = 0.5 instead of 1 – log 2 = 0.6990.
- Counting digits as the integer part only (it is integer part + 1).
- Mixing bases when combining logs.

Points to remember

- $\log_a x = y \Leftrightarrow a^y = x$.
- Product → add, quotient → subtract, power → multiply.
- $\log_a b \times \log_b a = 1$.
- a^(log$_a$ x) = x.
- log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990.

Speed and accuracy

- Rewrite numbers as powers of a common prime.
- Spot chains and telescoping sums before calculating.
- Test options in equations – and check the domain.
- Keep the standard log values at your fingertips.

Preparation strategy

- Learn the definition and the three laws until they are automatic.
- Practise on LearnFrenzy in order: Easy → Medium → Hard → Exam Pattern Mix → Data Sufficiency.
- Aim for 30–45 seconds per log question.
- Keep an error log of invalid-law and domain slips.
- Read the Quick Revision Sheet (next page) the night before the exam.

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6 Quick revision sheet

Everything on one page for the night before the exam

Formulas

Definition	$\log_a x = y \Leftrightarrow a^y = x$	$\log_a 1$ / $\log_a a$	0 / 1
Product	log m + log n	Quotient	log m − log n
Power	n log m	Change of base	log b/log a
Reciprocal	$\log_a b \cdot \log_b a = 1$	Chain	$\log_a b \cdot \log_b c = \log_a c$
Base / number powers	$(n/m) \log_a b$	Identity	$a^{(\log_a x)} = x$
Swap	a^(log b) = b^(log a)	Digits in N	⌊log N⌋ + 1
Zeros after point	k for log = −k.xxx	Domain	base > 0, ≠ 1; argument > 0

Shortcut tricks

- One base for everything.
- Chains: first base, last number.
- Telescoping: last ÷ first.
- log 5 = 1 − log 2.
- Digits = integer part + 1.
- Always check the domain.

Must-know numbers

Log values	log 2 = 0.3010 · log 3 = 0.4771 · log 5 = 0.6990 · log 7 = 0.8451
Built values	log 4 = 0.6020 · log 6 = 0.7781 · log 8 = 0.9030 · log 9 = 0.9542 · log 12 = 1.0791

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7 A message from LearnFrenzy

For everyone preparing for placements, competitive and government exams

Keep going – you are closer than you think

Success in examinations is not just about how many hours you study. It is about how effectively you understand concepts, practise questions, and improve yourself every day.

Every formula you learn, every problem you solve, and every mistake you correct brings you one step closer to your goal.

Whether you are preparing for campus placements, competitive examinations, or government jobs, remember that consistency, confidence, and smart preparation make a real difference.

Don’t just memorize formulas. Understand the concepts, practise smart techniques, and develop the confidence to solve problems independently.

Your hard work today can create the opportunities you dream of tomorrow.

— Team LearnFrenzy

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