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QUANTITATIVE APTITUDE · FREE STUDY MATERIAL

Ratio and Proportion

Formulas & Shortcut Tricks

Every important formula, shortcut and exam trick for Ratio and Proportion in one place – with solved examples, common mistakes and a one-page revision sheet.

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USEFUL FOR

- Bank PO & Clerk (IBPS, SBI, RBI)
- SSC CGL / CHSL / MTS
- Railways (RRB NTPC, Group D)
- Campus placements (TCS, Infosys, Wipro, Accenture)
- UPSC CSAT & state PSC exams

Practise Ratio and Proportion free on LearnFrenzy – questions with instant explanations →

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1 Topic introduction

What it is, why it matters and how it is asked

A ratio compares two quantities of the same kind by division: a : b means a/b. A ratio has no unit, and multiplying or dividing both terms by the same number does not change it – 12 : 18 is the same as 2 : 3. A proportion says that two ratios are equal, a : b = c : d.

Ratios are the backbone of sharing problems, partnerships, mixtures, ages, speeds and work. Once you think of a ratio as a number of equal “parts”, most questions become: find the value of one part, then multiply.

Why it matters in exams

- Bank, SSC and railway papers ask 2–4 ratio questions and use ratios in data interpretation.
- Placement tests combine ratios with incomes, coins, mixtures and variation.
- Partnership, mixtures and alligation, ages and time-and-work all use the same part method.

Where it is used

- Sharing money, profit or property in agreed proportions.
- Recipes, mixtures, map scales and model sizes.
- Comparing incomes, prices, populations and speeds.

Common question patterns

Combining ratios
A : B and B : C to A : B : C, or a long chain to A : D.

2A = 3B = 4C
Turn equal products into a ratio.

Proportionals
Fourth, third and mean proportional.

Dividing a sum
In a given ratio, an inverse ratio or with conditions.

Make a ratio
Add or subtract a number to reach a new ratio.

Income and savings
Incomes and expenditures in ratios, equal savings.

Variation
Direct, inverse, value $\propto$ weight^2, fixed + variable cost.

Mixtures and coins
Change the ratio of a mixture; coins in a ratio.

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2 Important formulas

Cards marked MOST USED are the ones to revise first

Kinds of ratios

Name	Of a : b	Example (2 : 3)
Duplicate	$a^2 : b^2$	4 : 9
Sub-duplicate	$\sqrt{a} : \sqrt{b}$	$\sqrt{2} : \sqrt{3}$
Triplicate	$a^3 : b^3$	8 : 27
Sub-triplicate	$\sqrt[3]{a} : \sqrt[3]{b}$	$\sqrt[3]{2} : \sqrt[3]{3}$
Inverse (reciprocal)	b : a	3 : 2
Compound of a : b and c : d	ac : bd	with 4 : 5 → 8 : 15

Compare two ratios by cross-multiplying: a : b > c : d exactly when ad > bc.

Combining ratios

A : B and B : C → A : B : C

MOST USED

Make the common term (B) equal using the LCM, then read off all three

Scale each ratio so that B has the same value in both.

EXAMPLE A : B = 2 : 3 and B : C = 4 : 5 → 8 : 12 and 12 : 15 → **8 : 12 : 15**.

Long chains

A : D = (A/B) × (B/C) × (C/D)

Multiply the ratios as fractions – the middle terms cancel.

EXAMPLE Compound of 2 : 3, 4 : 5 and 15 : 16 → (2 × 4 × 15) : (3 × 5 × 16) = 120 : 240 = **1 : 2**.

Equal multiples: 2A = 3B = 4C

MOST USED

A : B : C = 1/2 : 1/3 : 1/4 = (LCM ÷ 2) : (LCM ÷ 3) : (LCM ÷ 4)

The larger the multiplier, the smaller the quantity.

EXAMPLE 2A = 3B = 4C → LCM 12 → **6 : 4 : 3**.

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Proportion

Product of extremes = product of means

MOST USED

$$a : b = c : d \Leftrightarrow a \times d = b \times c$$

a and d are the extremes, b and c the means.

EXAMPLE Fourth proportional to 4, 9, 12 → d = 9 × 12 ÷ 4 = **27**.

Third and mean proportional

MOST USED

Third proportional of a, b = b^2/a; mean proportional of a, b = $\sqrt{(ab)}$

In a : b = b : c, b is the mean and c the third proportional.

EXAMPLE Mean proportional of 9 and 16 = $\sqrt{144}$ = **12**; third proportional of 4 and 10 = 100/4 = 25.

Componendo and dividendo

If $a/b = c/d$, then $(a + b)/(a - b) = (c + d)/(c - d)$

Works in both directions – very useful to remove sums and differences.

EXAMPLE $(x + y)/(x - y) = 7/3 \rightarrow x/y = (7 + 3)/(7 - 3) =$ **5 : 2**.

Dividing and changing

Dividing a sum in a ratio

MOST USED

Share = total × (its part/sum of parts)

Find the value of one part first.

EXAMPLE ₹1,260 in 2 : 3 : 4 → one part = 140 → **₹280, ₹420, ₹560**.

Dividing in an inverse ratio

Divide in 1/a : 1/b : 1/c, written in whole numbers with the LCM

Inverse of 2 : 3 : 4 is 6 : 4 : 3.

EXAMPLE ₹1,300 in the inverse ratio of 2 : 3 : 4 → 6 : 4 : 3 → **₹600, ₹400, ₹300**.

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Adding the same number to make a new ratio

MOST USED

$$(a + x) : (b + x) = c : d \rightarrow x = (bc - ad)/(d - c)$$

Cross-multiply and solve one linear equation.

EXAMPLE Add x to 7 and 11 to get 2 : 3 → x = (22 − 21)/(3 − 2) = **1**.

Increase each term by a percentage

$$\text{New ratio} = a(1 + p/100) : b(1 + q/100) : \ldots$$

Multiply each term by its own factor, then simplify.

EXAMPLE Seats 3 : 4 : 5 increased by 40%, 50%, 75% → 4.2 : 6 : 8.75 = **84 : 120 : 175**.

Incomes, variation and mixtures

Income, expenditure and savings

MOST USED

$$\text{Incomes } ak, bk; \text{ expenditures } cm, dm; \text{ savings} = \text{income} - \text{expenditure for each person}$$

Two unknowns (k and m) and two equations from the savings.

EXAMPLE Incomes 5 : 4, expenditures 3 : 2, each saves ₹2,000 → 5k − 3m = 4k − 2m = 2,000 → incomes **₹5,000 and ₹4,000**.

Direct and inverse variation

MOST USED

$$\text{Direct: } y = kx \ (y_1/x_1 = y_2/x_2). \ \text{Inverse: } y = k/x \ (x_1y_1 = x_2y_2). \ \text{Joint: } y = kxz$$

Find k from one pair of values, then use it.

EXAMPLE 12 workers finish a job in 15 days; 18 workers take 12 × 15 ÷ 18 = **10 days**.

Fixed cost plus variable cost

$$\text{Total} = \text{fixed} + k \times \text{quantity} \rightarrow \text{use two cases to find both}$$

The difference between two totals removes the fixed part.

EXAMPLE 20 students ₹14,000, 30 students ₹19,000 → ₹500 each + ₹4,000 fixed → 45 students **₹26,500**.

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Changing the ratio of a mixture

> **The part that is not added stays the same – write the new ratio around it**

Adding water does not change the amount of milk.

EXAMPLE 40 L with milk : water = 5 : 3 → milk 25, water 15 → for 1 : 1 add **10 L** water.

Repeated replacement

> **Pure liquid left = $V \times (1 - x/V)^n$**

Each time, the same fraction of what is left is removed.

V – volume of the vessel **x** – amount taken out each time **n** – number of times

EXAMPLE 40 L milk, 4 L replaced by water twice → 40×0.9^2 = **32.4 L** milk.

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3 Shortcut tricks & smart techniques

Each trick says when to use it – that matters as much as the trick

Equal-term method for chains

MOST USED

USE WHEN Several ratios share terms (A : B, B : C, C : D).

	A	B	C	D
A : B = 3 : 4	3	4		
B : C = 6 : 7		6	7	
C : D = 14 : 15			14	15
Multiply down	3 × 6 × 14	4 × 6 × 14	4 × 7 × 14	4 × 7 × 15

EXAMPLE A : D = (3 × 6 × 14) : (4 × 7 × 15) = 252 : 420 = **3 : 5**.

Value of one part

USE WHEN Any sharing question with a total, a difference or one known share.

1. Find how many parts the known amount represents.
2. Divide to get one part, then multiply for the share asked.

EXAMPLE Ratio 5 : 8 and the shares differ by ₹450 → 3 parts = 450 → larger share 8 × 150 = **₹1,200**.

Same amount added: the difference is unchanged

USE WHEN The same number is added to (or taken from) both terms.

1. Write both ratios with the same difference of terms.
2. The change in a term equals the number added.

EXAMPLE 3 : 5 becomes 5 : 7 after adding x to both (difference 2 in each) → each term rose by 2 parts; if the numbers were 9 and 15, x = **6**.

Loss when a precious stone breaks

MOST USED

USE WHEN Value varies as the square of the weight and the stone breaks into pieces.

1. Pieces in ratio a : b keep $(a^2 + b^2)/(a + b)^2$ of the value.
2. Loss fraction for two pieces = $2ab/(a + b)^2$.

EXAMPLE ₹36,000 diamond broken in 1 : 2 → loss = 4/9 × 36,000 = **₹16,000**.

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4 Solved examples

The full method on the left, the exam shortcut on the right

1 CHAIN OF RATIOS

If A : B = 3 : 4, B : C = 6 : 7 and C : D = 14 : 15, find A : D.

STEP BY STEP

1. A/D = (A/B) × (B/C) × (C/D).
2. = 3/4 × 6/7 × 14/15 = 252/420.
3. = 3/5 → A : D = 3 : 5.

SHORTCUT

Cancel before multiplying: (3 × 6 × 14) ÷ (4 × 7 × 15) = 3/5.

Why it works: B and C appear once on top and once below, so they cancel.

Answer: 3 : 5

2 EQUAL MULTIPLES

If 3A = 4B = 6C, find A : B : C.

STEP BY STEP

1. Let 3A = 4B = 6C = k.
2. A = k/3, B = k/4, C = k/6.
3. Take k = 12: A : B : C = 4 : 3 : 2.

SHORTCUT

LCM of 3, 4, 6 = 12 → 12/3 : 12/4 : 12/6 = 4 : 3 : 2.

Why it works: Equal products mean each quantity is inversely proportional to its multiplier.

Answer: 4 : 3 : 2

3 CONDITIONAL SHARING

₹7,200 is shared among A, B and C so that A gets half of B's share and B gets one-third of C's share. Find C's share.

STEP BY STEP

1. Let C = 6 parts → B = 2 parts → A = 1 part.
2. Total parts = 9 → one part = 800.
3. C = 6 × 800 = ₹4,800.

SHORTCUT

A : B : C = 1 : 2 : 6 → C = 7,200 × 6/9 = 4,800.

Why it works: Start from the person everyone else is compared with.

Answer: ₹4,800

4 MEAN PROPORTIONAL

Find the mean proportional between 0.08 and 0.18.

STEP BY STEP

1. Mean proportional = $\sqrt{}$(0.08 × 0.18).
2. = $\sqrt{}$0.0144 = 0.12.

SHORTCUT

$\sqrt{}$(8 × 18) = 12, decimal places 4 → 2 → 0.12.

Why it works: In 0.08 : x = x : 0.18, x^2 = 0.08 × 0.18.

Answer: 0.12

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5 MAKE PROPORTIONAL

What number must be subtracted from each of 23, 30, 57 and 78 so that the remainders are in proportion?

STEP BY STEP

1. $(23 - x)(78 - x) = (30 - x)(57 - x)$.
2. $1{,}794 - 101x = 1{,}710 - 87x$.
3. $84 = 14x \rightarrow x = 6$.

SHORTCUT

Test options: $17 : 24 = 51 : 72$? $17 \times 72 = 1{,}224 = 24 \times 51$ ✓ for $x = 6$.

Why it works: Product of extremes equals product of means.

Answer: 6

6 INCOME AND SAVINGS

The incomes of A and B are in the ratio 7 : 5 and their expenditures in the ratio 3 : 2. Each saves ₹6,000. Find A's income.

STEP BY STEP

1. $7k - 3m = 6{,}000$ and $5k - 2m = 6{,}000$.
2. Multiply: $14k - 6m = 12{,}000$ and $15k - 6m = 18{,}000$.
3. Subtract: $k = 6{,}000 \rightarrow$ A's income = 42,000.

SHORTCUT

Eliminate m with multipliers 2 and 3, then read k.

Why it works: Equal savings give two equations in the two unknown parts.

Answer: ₹42,000

7 VARIATION

The value of a diamond varies as the square of its weight. A diamond worth ₹1,44,000 breaks into three pieces whose weights are in the ratio 1 : 2 : 3. What is the loss?

STEP BY STEP

1. Whole weight = 6 units → value $\propto$ 36.
2. Pieces: $1^2 + 2^2 + 3^2 = 14 \rightarrow$ value = $14/36 \times 1{,}44{,}000 = 56{,}000$.
3. Loss = 1,44,000 − 56,000 = 88,000.

SHORTCUT

Loss fraction = $(36 - 14)/36 = 22/36 \rightarrow 22/36 \times 1{,}44{,}000$ = ₹88,000.

Why it works: Squares of the parts add up to less than the square of the whole.

Answer: ₹88,000

8 FIXED + VARIABLE

A taxi charges ₹180 for 10 km and ₹252 for 16 km (a fixed charge plus a rate per km). What is the fare for 25 km?

STEP BY STEP

1. Extra 6 km cost ₹72 → ₹12 per km.
2. Fixed charge = 180 − 120 = ₹60.
3. 25 km: $60 + 25 \times 12$ = ₹360.

SHORTCUT

Rate = 72/6 = 12; fare = 60 + 300 = 360.

Why it works: Subtracting the two fares removes the fixed charge.

Answer: ₹360

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9 MIXTURE

A vessel holds 84 litres of milk and water in the ratio 5 : 2. How much water must be added to make the ratio 3 : 2?

STEP BY STEP

1. Milk = 60 L, water = 24 L.
2. For 3 : 2 with 60 L milk, water = 40 L.
3. Water to add = 40 − 24 = 16 L.

SHORTCUT

Milk fixed at 60 = 3 parts → 2 parts = 40 → add 16.

Why it works: Only water is added, so the milk stays 60 L.

Answer: 16 litres

10 PERCENTAGE CHANGE

Seats for Maths, Physics and Biology are in the ratio 5 : 7 : 8. They are increased by 40%, 50% and 75% respectively. What is the new ratio?

STEP BY STEP

1. New: 5 × 1.4 = 7, 7 × 1.5 = 10.5, 8 × 1.75 = 14.
2. 7 : 10.5 : 14 = 14 : 21 : 28.
3. = 2 : 3 : 4.

SHORTCUT

Multiply each term by its factor, double to clear decimals, divide by 7.

Why it works: Each subject grows at its own rate, so each term gets its own factor.

Answer: 2 : 3 : 4

11 COINS

A box has ₹2, ₹1 and 50-paise coins in the ratio 3 : 5 : 8. The total value is ₹285. How many 50-paise coins are there?

STEP BY STEP

1. One set (3, 5, 8 coins) is worth 6 + 5 + 4 = ₹15.
2. Sets = 285 ÷ 15 = 19.
3. 50-paise coins = 8 × 19 = 152.

SHORTCUT

Value per set first, then multiply the count.

Why it works: The ratio is of numbers of coins, not of their values.

Answer: 152

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5 Important exam tips

Patterns, traps and habits that save marks

Frequently asked question patterns

- Combining A : B and B : C, or chains to A : D.
- “2A = 3B = 4C” type ratios.
- Fourth, third and mean proportional.
- Dividing money directly, inversely or with conditions.
- Incomes and expenditures with equal savings.
- Variation: inverse (workers–days), value $\propto$ weight^2, fixed + variable cost.
- Mixtures, replacement, coins and percentage changes in ratios.

Common mistakes to avoid

- Writing 2A = 3B as A : B = 2 : 3 – it is 3 : 2.
- Combining A : B and B : C without making B equal first.
- Treating a ratio of coins as a ratio of their values.
- Adding the same number to both terms and expecting the ratio to stay the same.
- Forgetting to simplify the final ratio.
- In mixtures, changing the quantity that was not added.

Points to remember

- a : b = c : d $\Leftrightarrow$ ad = bc.
- Mean proportional = $\sqrt{(ab)}$; third proportional = b^2/a.
- Inverse of a : b : c is bc : ca : ab.
- Componendo–dividendo: $(a + b)/(a - b) = (c + d)/(c - d)$.
- Loss on breaking (value $\propto$ weight^2) into a : b = $2ab/(a + b)^2$.
- Map scale 1 : n – 1 cm on the map is n cm on the ground (4 cm at 1 : 50,000 = 2 km).

Speed and accuracy

- Always find the value of one part.
- Cancel common factors before multiplying chains.
- Use option testing for “make proportional” questions.
- Keep the unchanged quantity fixed in mixture questions.

Preparation strategy

- Learn combining ratios and the part method first – everything else builds on them.
- Practise on LearnFrenzy in order: Easy → Medium → Hard → Exam Pattern Mix → Data Sufficiency.
- Aim for 40 seconds per ratio question.
- Keep an error log of inverted ratios (2A = 3B traps).
- Read the Quick Revision Sheet (next page) the night before the exam.

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6 Quick revision sheet

Everything on one page for the night before the exam

Formulas

Proportion	$a : b = c : d \Leftrightarrow ad = bc$	Fourth proportional	bc/a
Third proportional	b^2/a	Mean proportional	$\sqrt{(ab)}$
Duplicate / sub-dup.	$a^2 : b^2$ / $\sqrt{a} : \sqrt{b}$	Compound	$ac : bd$
A : B : C	make B equal (LCM)	2A = 3B = 4C	6 : 4 : 3
Share	total × part/sum	Add x to make c : d	$x = (bc - ad)/(d - c)$
Componendo–dividendo	$(a + b)/(a - b)$	Direct / inverse	$y = kx$ / $xy = k$
Fixed + variable	$F + kx$	Replacement	$V(1 - x/V)^n$
Break ($\propto w^2$) a : b	loss $2ab/(a + b)^2$	Map scale 1 : n	1 cm = n cm

Shortcut tricks

- One part first, then multiply.
- Chains: multiply, cancel middles.
- kA = … → divide LCM by multipliers.
- Coins: value of one set.
- Mixtures: keep the unchanged part.
- Same number added → same difference.

Must-know numbers

Inverse ratios	1:2:3 → 6:3:2 · 2:3:4 → 6:4:3 · 3:4:5 → 20:15:12
Squares and cubes	2:3 → 4:9 · 8:27 · 3:4 → 9:16 · 27:64
Units	1 km = 1,000 m = 1,00,000 cm · 1 kg = 1,000 g · ₹1 = 100 paise

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7 A message from LearnFrenzy

For everyone preparing for placements, competitive and government exams

Keep going – you are closer than you think

Success in examinations is not just about how many hours you study. It is about how effectively you understand concepts, practise questions, and improve yourself every day.

Every formula you learn, every problem you solve, and every mistake you correct brings you one step closer to your goal.

Whether you are preparing for campus placements, competitive examinations, or government jobs, remember that consistency, confidence, and smart preparation make a real difference.

Don't just memorize formulas. Understand the concepts, practise smart techniques, and develop the confidence to solve problems independently.

Your hard work today can create the opportunities you dream of tomorrow.

— Team LearnFrenzy

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