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REASONING ABILITY

LOGICAL VENN DIAGRAMS

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Logical Venn Diagrams

Logical Venn Diagrams

Venn diagrams are those pictures (diagrams) made of geometric shapes in which a logical relationship is shown between different groups or sets. Questions based on Venn diagrams are often asked in mental ability tests with the aim of testing the ability to understand a particular group well, establish its relationship and interpret it diagrammatically.

In this chapter we will learn about Venn diagrams and solving questions related to them.

Before studying Venn Diagram, let us know some basic points of Set Theory.

Set

A well-defined group or collection of different objects is called a set. The objects that form a set are called elements. Elements can be anything, such as digits, numbers, letters, words, people, any object, any other set or anything that can be defined. Sets are represented by capital letters of the English alphabet and each element of the set is written in curly brackets { }.

For example, the set of all natural numbers

$$N = \{1, 2, 3, 4, ...\infty\}$$

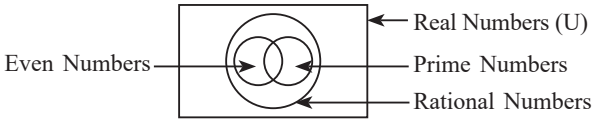
The set of all even numbers

$$E = \{2, 4, 6, ... 2n\}, n \in N$$

Universal Set

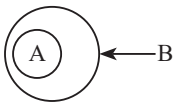
A set in which all the elements of the given different sets are present is called a universal set. It is represented by 'U'.

For example, the set of all even numbers, the set of all prime numbers and the universal set of all rational numbers will be real numbers.



Sub Set

Set A will be called a subset of set B, if all the elements of A are present in B. It is denoted by '⊂'.



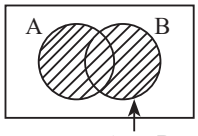
For example: If A = {2, 6}, B = {1, 2, 3, 4, 5, 6, 7, 8} It is clear here that the elements '2' and '6' of A are also present in set B. Hence A will be a subset of B.

That is, $A \subset B$

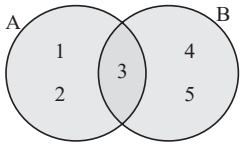
Operations of Sets

1. Union of Sets:

If there are any two sets A and B, then A ∪ B (A union B) will be a set in which every element of A and B exists.

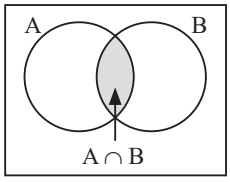


For example: A = {1, 2, 3}
B = {3, 4, 5}
A ∪ B = {1, 2, 3, 4, 5}

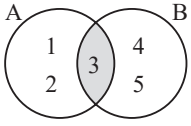


2. Intersection of Sets

If there are two sets A and B, then A ∩ B (A intersection B) will be a set which will contain only those elements which are present in both the sets (A and B).



For example: A = {1, 2, 3}
B = {3, 4, 5}
A ∩ B = {3}



Actually, representing a given set or sets through a geometric method is called a Venn diagram. A mathematician named Euler first started the method of giving geometric shapes to sets. Later this method was developed by John Venn and it is called a Venn diagram after his name.

Understanding of Venn Diagram

When we represent a group or set of objects of one type through a diagram, we use circles. But sometimes we also use rectangles (▭) to represent universal sets.

For example, if we want to represent the set of humans and men through a Venn diagram, then we will represent it in the following manner.

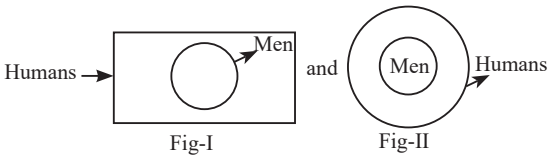


Fig-I Fig-II

Here, the relation between humans and men is shown by two figures. We know that all men are humans i.e. men are a subset of humans. So, we have represented humans by a big circle or rectangle, while men by a small circle which is just inside the first big circle or rectangle.

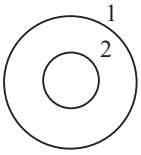
To understand Venn diagrams better, we divide them into two parts.

1. When the sets (groups) are only two.
2. When the groups are three or more than three.

1. When there are only two sets

When we establish relations between only two sets, we usually get the following diagrams.

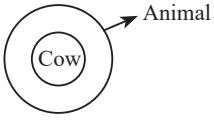
Case-1



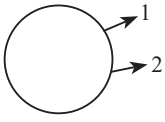
Circle 2 is completely contained in Circle 1 i.e. all the elements of Circle 2 are present in Circle 1 or (2 ⊂ 1) i.e. set 2 is a subset of set 1.

For example, if we want to show the relation between animals and cow, then

'Cow ⊂ Animal' will be written.



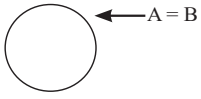
Case-2



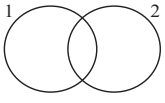
It is clear from the diagram of case 2 that here both the diagrams are exactly the same i.e. every element of set 1 is in set '2' and every element of set '2' is also in set 1.

Set 1 = Set 2

For example: if set A = {2, 4, 6} and set B are all those even numbers which are less than 7.

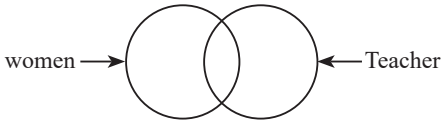


Case-3



This diagram means that neither of the two groups is completely included in the other group. But some part of both groups are the same.

For example: group of women and teachers



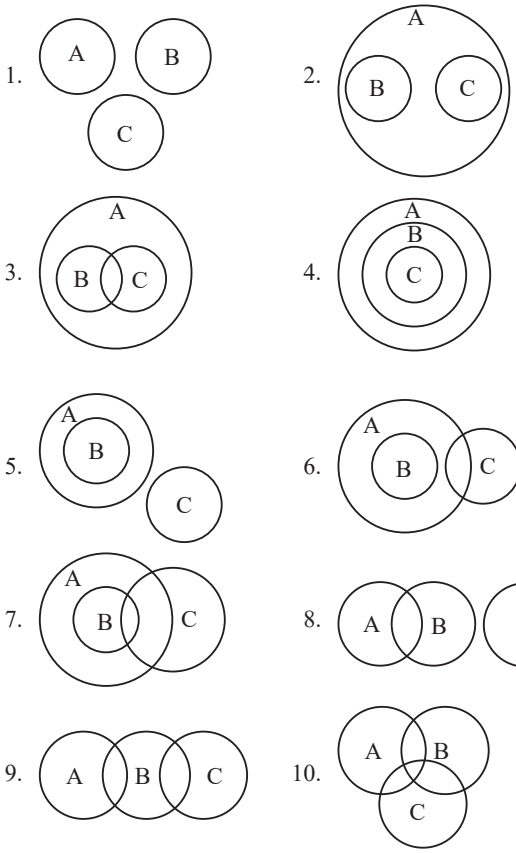
Some women can also be teachers and some women can also be among the teachers.

Case-4 : When none of the two groups is fully or partially included in each other, that is, there is nothing common between the two groups.

For example: group of pens and pencils

2. When three groups are present

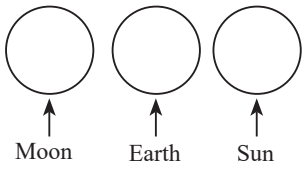
When three groups are present in the question, the following type of Venn diagram can be made.



Let us understand some of these with the help of examples.

Case-1 : When each of the three types of groups is completely different; no part of any group lies either fully or partially in any other group.

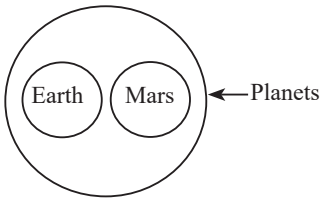
For example: Moon, Earth, Sun



All three are celestial bodies but no two are similar to each other and no two have anything in common.

Case-2 : When two different types of groups are completely included in a third broader group.

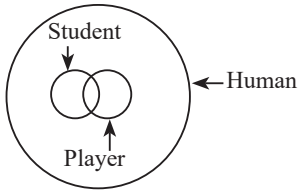
For example: Planets, Earth, Mars



Planet is the name of a broad group which includes Earth and Mars whereas Earth and Mars have nothing in common.

Case-3 : In this type of Venn diagram, there are two groups which are completely included in a third group and both the groups are partially included in each other.

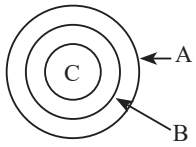
For example: group of humans, students and players.



It is clear that both students and players will be humans. But some players can be students and some students can be players.

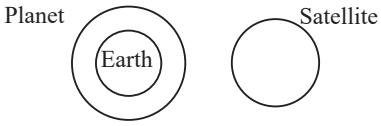
Case-4 : In this type of diagram, one group is completely contained within the second group while the third group is completely contained within the second group.

For example, if we represent animals by A, pets by B and cow by C, then the following diagram will be obtained.



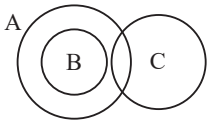
Case-5 : When there are two groups such that one of them is completely included in the other group, while the third group is completely different from both the groups.

For example: Planets, Satellites, Earth



Case-6 : When one of the two groups is completely included in the other group while it is completely different from the third group but the third group is partially included in the larger group.

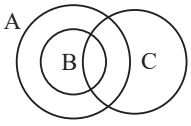
For example: If we represent all men by A, all fathers by B and unmarried people by C, then we will get the following Venn diagram.



It is clear here that all fathers must be men and there cannot be any unmarried father, although it is possible that he may be a woman.

Case-7 : When a group is completely included in the other group and the third group is partially related to both the groups.

For example, if we denote women by A, mothers by B and teachers by C, then the following diagram will be obtained.



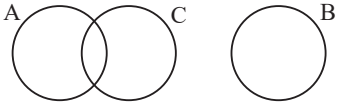
Mother can only be a woman so she is inside circle A whereas teachers can be male as well as female and female teachers may or may not be mothers.

Case-8 : When two groups are partially included in each other while the third group is completely different from both.

For example: Some boys play hockey.

No girl plays hockey.

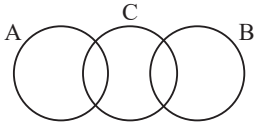
If boys are represented by A, girls by B and hockey players by C, then the following diagram will be obtained.



Here, apart from boys, other people like elderly people may also play hockey but girls are not included in this.

Case-9 : When any two of the three groups are partially included in each other while the third group is partially included in one of them and has no connection with the other group.

For example, if boys are represented by A, girls are represented by B, while teachers are represented by C, then the following diagram will be obtained.



Here boys and girls cannot be included in any way whereas teachers can be boys, girls and any other man, woman and old person also.

Case-10: When the three groups are related in such a way that every two groups have something in common and also the three groups are partially included in each other.

For example, let us assume that there are 3 groups A, B and C. Their elements are as follows-

$A = \{a, b, c, d\}$

$B = \{b, c, e, f\}$

$C = \{b, d, e, g\}$

Here b is common in all three groups (sets).

That is, $A \cap B \cap C = \{b\}$

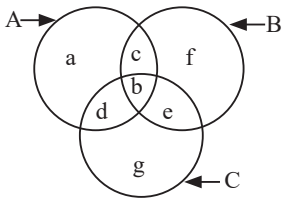
Also, some elements are common in every two sets.

$A \cap B = \{b, c\}$

$B \cap C = \{b, e\}$

$C \cap A = \{b, d\}$

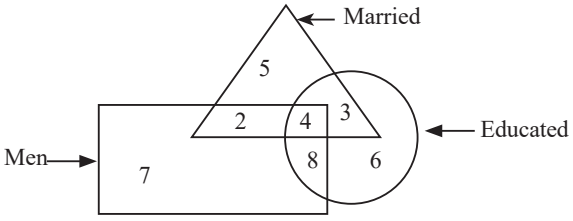
Their Venn diagram will be made as follows.



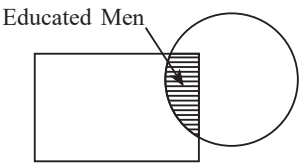
Applications of Venn Diagram

So far we have learnt to make different types of Venn diagrams. Some questions are asked in the question paper in which a Venn diagram is given and some data/information is also given along with it. On the basis of which the answer has to be given by understanding the Venn diagram.

For example, in the Venn diagram given below, married people are represented by triangle (Δ), educated people are represented by circle (○) and men are represented by rectangle (▭). Then tell how many men are there who are educated but not married?

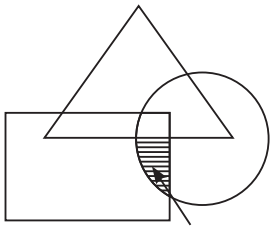


Sol. If we draw a Venn diagram of only males and educated people then we will get the following diagram.



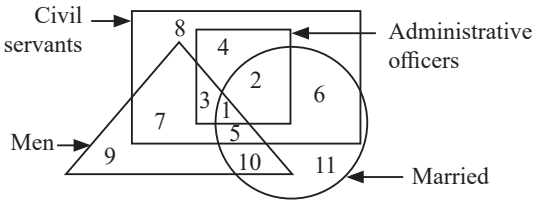
To remove men who are not married we will not take the triangle part.

This way we will have this Venn diagram.



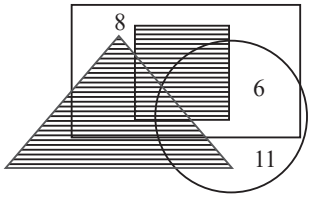
From the picture, the number of such men is 8.

Direction: Study the diagram given below carefully and answer the given questions. If in the diagram rectangle (▭) represents all civil servants, square (□) represents administrative officers, triangle (Δ) represents men and circle (○) represents married people.



1. How many civil servants are there who are neither administrative officers nor men?

Sol. While solving such questions, we remove the information which we need as 'no' in the answer. So leaving out men and administrative officers,



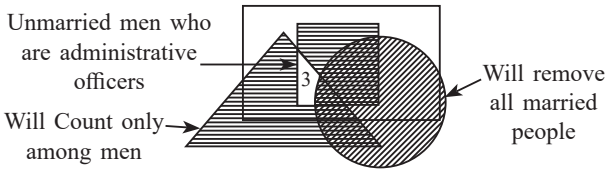
Now we are left with only three numbers 8, 6, and 11. So now we see the information we need as 'yes'. We ignore all others.

Nothing was asked about married or unmarried, hence we will leave this also.

Therefore, the number of civil servants who are neither administrative officers nor male will be (8 + 6) = 14.

2. What will be the number of unmarried administrative officers who are men?

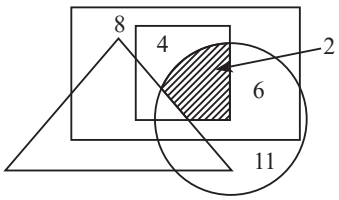
Sol.



It is clear from the diagram that the number of men who are unmarried and are administrative officers will be 3.

3. How many civil servants are married and administrative officers but are not men?

Sol.

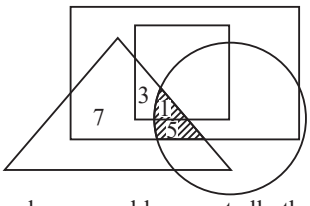


After removing all the men, whatever diagram remains, the common answer will be the one which is shown in shaded form.

So the required answer will be '2'.

4. How many married male civil servants are there?

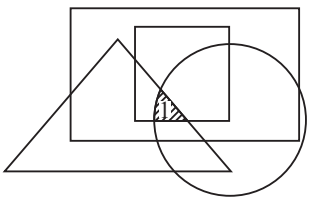
Sol.



Let us count only men and leave out all others. It is clear from the figure that the number of married and male civil servants is (1 + 5) = 6.

5. How many married men are there who are administrative officers?

Sol.



Take the common part of all. So the answer will be '1'.

Many times some questions are asked in the exam which are based on Venn diagrams, but Venn diagrams are not given. Although the questions can be solved without a Venn diagram, it is better to use a Venn diagram to solve the questions faster. Let us learn to solve the questions in both ways by taking an example.

For example:

(i) There are a total of 25 doctors and 35 women in a group. If there are 13 women doctors in the group, then how many people are there in the group?

Sol. Doctor, A = 25

Women, B = 35

Both woman and doctor, $A \cap B = 13$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Where $n(A)$ = number of elements of A

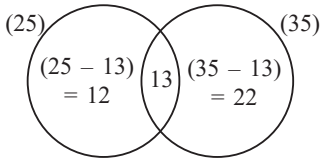
$n(B)$ = number of elements of B

$n(A \cap B)$ = elements which are common to both.

That means,

total number = 25 + 35 – 13 = 47

Second method



total number = 12 + 13 + 22 = 47

For example:

(ii) In a class 16 students failed in Mathematics, 18 students failed in English and 15 students failed in Science. While 7 students failed in both Mathematics and English, 8 students failed in both English and Science, 5 students failed in both Mathematics and Science and 3 students failed in all three subjects. Then tell how many students passed in the class, if there are a total of 50 students in the class.

Sol. Let us assume that, A = 16

B = 18

C = 15

and $A \cap B = 7$

$B \cap C = 8$

$C \cap A = 5$

$A \cap B \cap C = 3$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(B \cap C) + n(C \cap A)] + n(A \cap B \cap C)$$

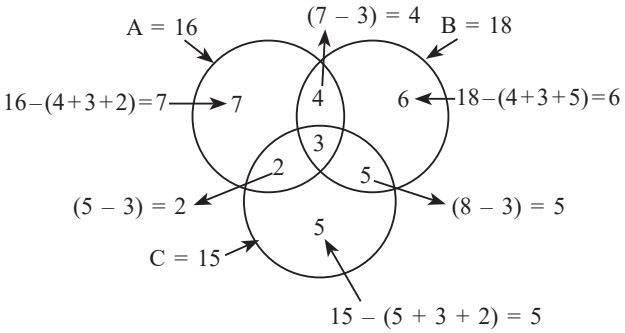
$\therefore A \cup B \cup C = (16 + 18 + 15) - (7 + 8 + 5) + 3$

$= 49 - 20 + 3 = 32$

That means there are 32 students who have failed in some subject or the other.

Number of students passed = 50 – 32 = 18

Or



By Venn diagram,

Total number of students who failed

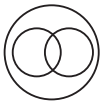
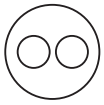
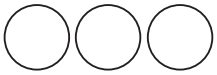
=7 + 4 + 6 + 2 + 3 + 5 + 5 = 32

$\therefore$ Total number of students who passed

= 50 – 32 = 18

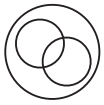
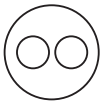
Example

1. Which figure represents the relationship between humans, trees and living beings?

(1) 
(2) 
(3) 
(4) 

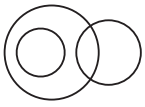
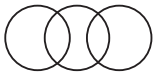
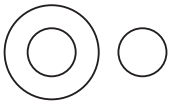
Sol. Here, living beings are a broad group, which includes both humans and trees, while there is no similarity between humans and trees. Hence, option (3) is correct.

2. Which figure shows the relation between circle, diameter and arc?

(1) 
(2) 
(3) 
(4) 

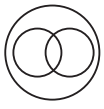
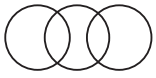
Sol. Diameter and arc are two different groups. But both are contained inside the circle. Hence, option (2) is correct.

3. Which figure correctly represents the relationship between student, graduate and singer?

(1) 
(2) 
(3) 
(4) 

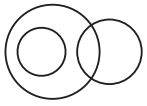
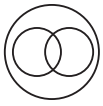
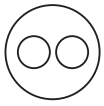
Sol. Any student can be a graduate and also a singer. Similarly, any graduate can be a student or a singer or both. There may be something common in every two groups and there may be something common in all three groups. Hence, option (1) is correct.

4. Which of the Venn diagrams given below correctly represents the relationship between tall men, black haired people and Indians?

(1) 
(2) 
(3) 
(4) 

Sol. All the three groups may or may not be partially included in each other. An Indian may or may not be a man. A man may or may not be black haired. A black haired person may or may not be an Indian. Hence option (4) is correct.

5. If all mangoes are fruits, all apples are also fruits then which of the following Venn diagrams can be possible?

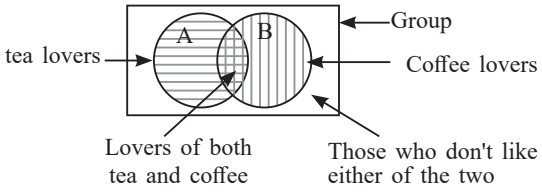
1. 
2. 
3. 
4. 

(1) Only 3
(2) 2 and 4
(3) 2 and 3
(4) 2, 3 and 4

Sol. It is clear in the question that all mangoes are fruits and all apples are also fruits i.e. circles of mango and apple will come inside the circle of fruit. But nothing is given about apples and mango. So we will make a Venn diagram avoiding the conclusion that apples and mango are different fruits. There (2), (3) and (4) all three can be possible. Hence, option (4) is correct.

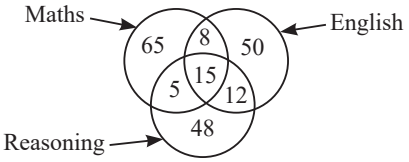
6. In a group, some people like tea and some like coffee. Some people like both tea and coffee. If there are some people in the group who do not like either tea or coffee, then what will be the Venn diagram for this situation?

Sol.



In the figure, a circle (1) represents people who like tea and another circle (2) represents people who like coffee. The common part of circle A and circle B (A ∩ B) represents people who like both coffee and tea. There are some people in the group who do not like either of the two, so they will be outside both the circles. Therefore, a universal set has been used to represent the entire group, which is made up of a rectangle.

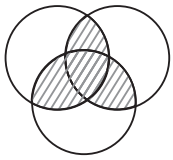
7. A class of 500 students had an exam in three subjects Maths, English and Reasoning. The diagram given below shows how many students failed in which subject. By studying the diagram carefully you have to find out how many percent of students failed in at least two subjects?



Sol. Total Students = 500

To calculate the percentage, we need only the number of students who fail in at least two subjects, i.e. they can fail

in two subjects as well as all three subjects. Therefore, we will not count all those students who fail in only one subject.



So taking only the shaded portion,

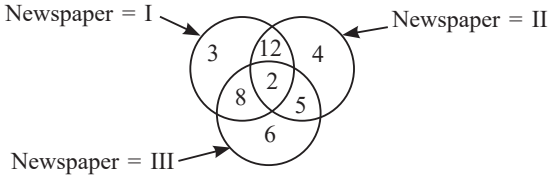
Students failed in at least two subjects

$= (8 + 15 + 5 + 12)$

$= 40$

$$\text{Percentage} = \frac{40}{500} \times 100 = 8 \text{ percent}$$

8. In the figure given below, the readers of three types of newspapers are represented by separate diagrams. This diagram has been made after surveying 50 people. You have to study these diagrams carefully and find out how many readers are there in a population of 50,000 people who read at least two types of newspapers?



Sol. Reading at least two newspapers means people who read two newspapers or three. Total number of such people

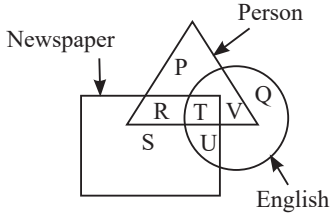
$= 12 + 2 + 8 + 5 = 27$

That is, out of 50 people, 27 people read at least 2 newspapers.

Similarly, population out of

$$50{,}000 = \frac{27}{50} \times 50{,}000 = 27000$$

9. Directions (Q. i-iv): In the picture given below some groups are shown by geometrical diagram. If '□' represents newspaper, 'Δ' represents person and 'O' represents English then answer the given questions.

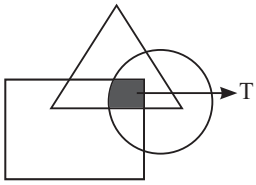


(i) On the basis of the diagram, what is the number of people reading English newspapers?

(1) T (2) R

(3) P (4) U

Sol. We have to find the part which is common in all the three parts.



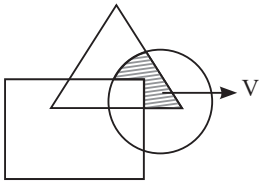
Hence, option (1) is correct.

(ii) How many people are there who know English but do not read newspapers?

(1) R (2) V

(3) T (4) U

Sol. Firstly we will remove the newspaper diagram and from the remaining part we will take only that part which is common between English and person.



Hence, option (2) is correct.

(iii) How many newspapers are there which are not in English?

(1) S

(2) R

(3) S + R

(4) S + R + P

Sol. First of all we remove the diagram of English. Now we count only the newspapers which are not in English. Here nothing is said about the person. So the newspapers which are not in English will be (S + R). Hence, option (3) is correct.

(iv) On the basis of the diagram, tell how many people read Hindi newspapers?

(1) P

(2) R

(3) R + P

(4) Cannot be determined.

Sol. In the question, the circle tells how much part is English. From this, we can find out how much part is not English but we cannot tell how much is Hindi because there is no information about Hindi. Hence, option (4) is correct.

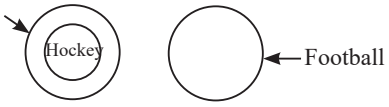
10. Consider the given statements.

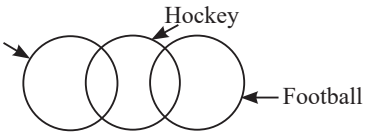
(i) Some cricket players play hockey.

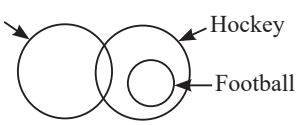
(ii) The one who plays football does not like cricket at all.

On the basis of the above information, test which Venn diagram can correctly depict the relationship between the two statements?

1. 

2. 

3. 

4. 

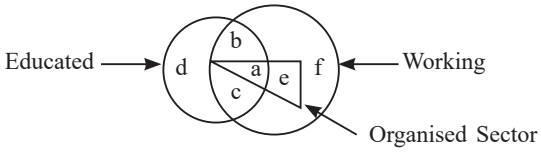
(1) Only 1 (2) 1 and 3

(3) 2 and 4 (4) All 1, 2, 3 and 4

Sol. According to statement (i) some cricketers play hockey i.e. there is a perfect or partial relationship between cricket and hockey.

According to statement (ii), both cricket and football are different. In each Venn diagram, no relation is shown between football and cricket. While in each diagram, it is included that some of the people who play cricket also play hockey. In this way, all the four diagrams can be possible. Hence, option (4) will be correct.

11. From the diagram given below find out the portion who are educated but working in unorganised sector?

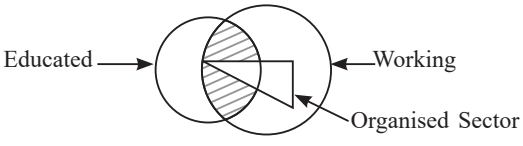


(1) b (2) d + a

(3) c + a (4) b + c

Sol. To find the correct answer, we will subtract the number (sector) who are working in the organized sector from those who are educated and employed.

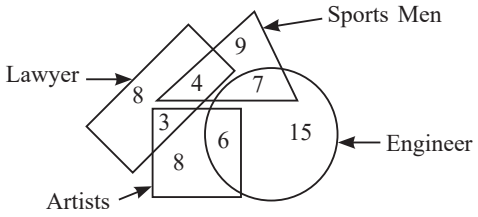
i.e. the area for the correct answer



Shaded area = b + c

So the required answer will be (4).

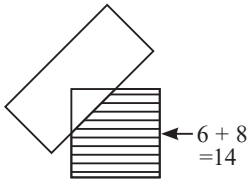
12. Study the diagram given below carefully and answer the questions based on it.



I. How many artists are not lawyers?

(1) 8 (2) 6

(3) 3 (4) 14



Sol. Here lawyers are represented by rectangle and artists by square, so leaving aside the other two shapes, those artists who are not lawyers will be identified as follows-

Hence, option (4) is correct.

II. How many engineers are there who are also sportsmen and artists?

(1) 6 (2) 7

(3) 13 (4) 0

Sol. It is clear from the diagram that there is nothing common between sportsmen and artists. Hence there will be no engineer who is both an artist and a sportsman. Hence, option (4) is correct.

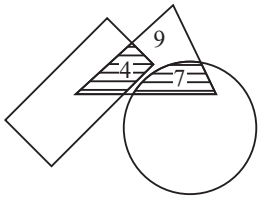
III. How many players are there who are either lawyers or engineers?

(1) 4 (2) 11

(3) 9 (4) 16

Sol. We have to find the number of players who are either lawyers or engineers.

From the diagram,



That is,4 + 7 = 11

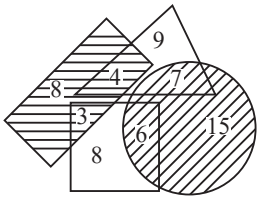
Hence, option (2) is correct.

IV. How many people are there in total who are either lawyers or engineers?

(1) 23 (2) 33

(3) 43 (4) 53

Sol. From the diagram,



Either a lawyer or an engineer,

That is n(lawyer) $\cup$ n(engineer)

$= (8 + 3 + 4) + (15 + 6 + 7)$

$= 15 + 28 = 43$

Hence, option (3) is correct.

13. There are 100 students in a class. In the examination of three subjects- Hindi, English and Science, 14 students failed in Hindi, 25 students failed in English and 22 students failed in Science. If a total of 60 students passed in the class and there are 3 students who failed in all three subjects, then tell how many students are there who failed in only two subjects?

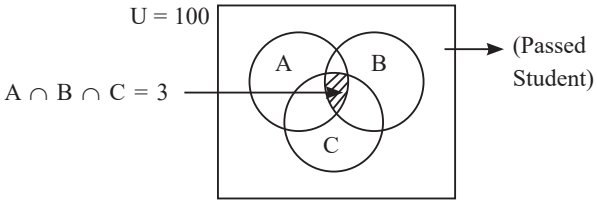
(1) 24 (2) 15

(3) 21 (4) 18

Sol. We cannot solve such type of questions directly through Venn diagrams.

Let us assume the set of students who failed in Hindi is A and the set of those who failed in English is B, while the set of those who failed in Science is C.

$A = 14, B = 25, C = 22$



$A \cup B \cup C = 100 - 60 = 40$

We know that,

$A \cup B \cup C = A + B + C - (A \cap B + B \cap C + C \cap A) + (A \cap B \cap C)$

$40 = 14 + 25 + 22 - x + 3$

(Let $A \cap B + B \cap C + C \cap A = x$)

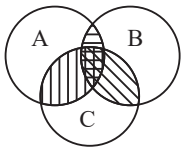
$40 = 61 - x + 3$

$x = 24$

Note: The important thing to note here is that in $x = 24$, 24 is the value of $(A \cap B + B \cap C + C \cap A)$ Whereas according to the question we have to find the number which represents the students who failed in only two subjects.

So, we have to find

$(A \cap B) \cup (B \cap C) \cup (C \cap A) - 3(A \cap B \cap C)$.



It is clear from the figure that we have counted the middle part three times.

So, from $(A \cap B) \cup (B \cap C) \cup (C \cap A)$ we subtract the middle part 3 times, which will leave us with only the part we need.

Students who fail in only two subjects

$= 24 - 3(3)$

$= 24 - 9 = 15$

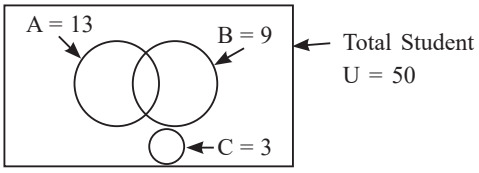
Hence, option (2) is correct.

14. In an examination of a class of 50 students, 13 students failed in Science, 9 students failed in English and 3 students failed in Sanskrit. None of the students who failed in Sanskrit failed in any other subject. If only 30 students passed in the class, then tell how many students are there who failed in two subjects?

(1) 6 (2) 7

(3) 5 (4) 9

Sol.



Passed students = 30

Students failed in some subject = 50 – 30 = 20

Failed in Sanskrit, did not fail in any other subject, hence failed in Science or English = 20 – 3 = 17

i.e. $A = 13$

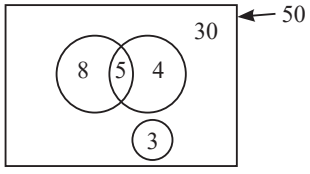
$B = 9$

$A \cup B = 17$

$A \cup B = A + B - (A \cap B)$

$17 = 13 + 9 - (A \cap B)$

$A \cap B = 22 - 17 = 5$



Hence, number of students who failed in two subjects = 5

Hence, option (1) is correct.

15. In a group of 30 people, 15 people like tea. 9 people like salty snacks and 5 people like to have biscuits with tea. No one likes biscuits and salty snacks together. If there are 8 people in the group who do not like to eat or drink anything, then tell how many people in the group take only tea?

(1) 6 (2) 7
(3) 8 (4) 9

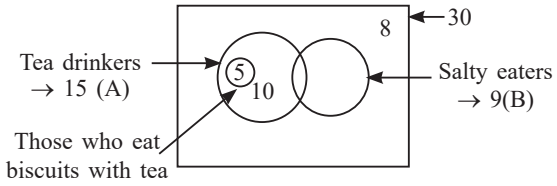
Sol. Number of people = 30

People who don't like anything = 8

People who like something or the other

$= 30 - 8 = 22 = (A \cup B)$

Since 5 people take biscuits with tea, biscuit eaters are included in tea drinkers.



$A \cup B = 22$

$A = 15$

$B = 9$

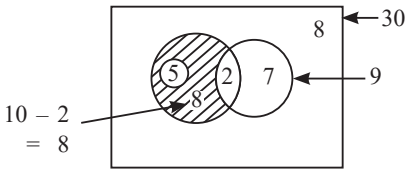
$A \cap B = (A + B) - (A \cup B)$

$= 15 + 9 - 22$

$= 24 - 22 = 2$

That means there are 2 people who drink tea and also eat snacks.

From the Venn diagram,

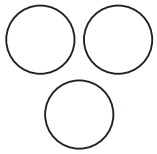


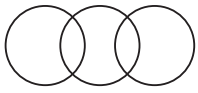
So the shaded part will be the part which represents those people who drink only tea and neither eat biscuits nor snacks.

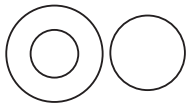
Hence, option (3) is correct.

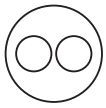
Practice Questions

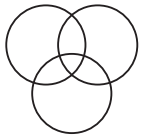
Directions (Q. 1-11): *Some Venn diagrams are given below which show the relationship of any group with another group. Identify the correct Venn diagram based on the given questions and choose the correct answer option.*

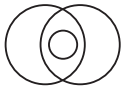
(1) 

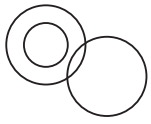
(2) 

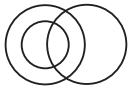
(3) 

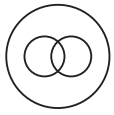
(4) 

(5) 

(6) 

(7) 

(8) 

(9) 

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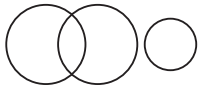
(11) 

1. Trees, Furniture, Wood
(1) (1) (2) (3)
(3) (10) (4) (4)

2. Paper, Pen, Ink
(1) (11) (2) (10)
(3) (5) (4) (2)

3. Page, Question Paper, book
(1) (4) (2) (2)
(3) (6) (4) (7)

4. Sun, Moon, Earth
(1) (1) (2) (4)
(3) (6) (4) (5)

5. Sun, Light, Heat
(1) (1) (2) (2)
(3) (4) (4) (5)

6. Sun, Moon, Stars
(1) (1) (2) (3)
(3) (4) (4) (6)

7. Satellite, Earth, Mars
(1) (1) (2) (3)
(3) (4) (4) (6)

8. Thief, Criminal, Judge
(1) (3) (2) (7)
(2) (1) (4) (11)

9. Rectangle, Square, Quadrilateral
(1) (10) (2) (4)
(3) (6) (4) (5)

10. Widow, Woman, Alcoholic
(1) (8) (2) (2)
(3) (10) (4) (9)

11. Punjab, Haryana, Chandigarh
(1) (2) (2) (7)
(3) (6) (4) (5)