LF LearnFrenzy

QUANTITATIVE APTITUDE · FREE STUDY MATERIAL

Problems on Numbers

Formulas & Shortcut Tricks

Every important formula, shortcut and exam trick for Problems on Numbers in one place – with solved examples, common mistakes and a one-page revision sheet.

WHAT'S INSIDE

1. Topic introduction
2. Important formulas
3. Shortcut tricks & smart techniques
4. Solved examples
5. Important exam tips
6. Quick revision sheet
7. A message from LearnFrenzy

USEFUL FOR

- Campus placements (TCS, Infosys, Wipro, Accenture, Cognizant)
- SSC CGL / CHSL / MTS
- Bank PO & Clerk (IBPS, SBI, RBI)
- Railways (RRB NTPC, Group D)
- UPSC CSAT & state PSC exams

Practise **Problems on Numbers** free on LearnFrenzy – questions with instant explanations →

© 2026 LearnFrenzy · Free for personal study

1 Topic introduction

What it is, why it matters and how it is asked

Problems on Numbers turn a sentence into an equation. “A number exceeds its reverse by 45” or “three consecutive even numbers add to 138” are puzzles whose real skill is translation: choose a sensible variable, write the condition correctly, and solve.

The algebra is rarely harder than one linear or quadratic equation. Marks are lost in misreading – “less than”, “of”, “reciprocal”, “reversed” – so this PDF starts with the words and ends with option-testing shortcuts.

Why it matters in exams

- Placement tests (TCS NQT, Infosys, Cognizant, Capgemini) open their arithmetic section with number puzzles.
- SSC and Railway papers ask digit-reversal, consecutive-number and wrong-operation questions.
- Bank exams use heads-and-legs, coins and negative-marking problems.

Where it is used

- Every word problem in later chapters – ages, work, profit – starts with this translation step.
- Exam scoring with negative marks, counting coins, sharing amounts.
- Puzzle and reasoning rounds in interviews.

Common question patterns

Two numbers
Sum, difference, ratio, product or squares given.

Consecutive
Consecutive, odd or even numbers with a given sum or product.

Digits
Two- and three-digit numbers, digit sums and reversals.

Fractions of a number
“3/5 of a number exceeds 2/7 of it by 33”.

Wrong operation
Divided instead of multiplied – find the number.

Heads and legs
Hens and cows, rabbits and chickens.

Marks and coins
Negative marking; coins in a ratio with a total value.

AP / GP triples
Three numbers in AP or GP with a sum and product.

© 2026 LearnFrenzy · Free for personal study

2 Important formulas

Cards marked MOST USED are the ones to revise first

Words into symbols

The question says	Write	With x = the number
7 more than / exceeds by 7	x + 7	“exceeds y by 7” → x − y = 7
7 less than	x − 7	not 7 − x
Twice / thrice	2x / 3x	“twice as much as y” → x = 2y
3/5 of	(3/5)x	“of” means multiply
Reciprocal	1/x	sum of reciprocals 1/x + 1/y
Consecutive / consecutive even or odd	x, x + 1, x + 2 / x, x + 2, x + 4	
Two-digit number with digits a, b	10a + b	reversed: 10b + a
Three-digit number	100a + 10b + c	reversed: 100c + 10b + a

Choose the variable for the quantity the question asks for, or for the smallest of the numbers – it usually keeps the equation free of fractions.

Two numbers

Sum and difference

MOST USED

Larger = (S + D)/2; Smaller = (S − D)/2

Add and halve for the larger, subtract and halve for the smaller.

S – sum **D** – difference

EXAMPLE Sum 74, difference 18 → **46 and 28**.

Ratio with a sum or difference

Numbers a : b → ak and bk; k = sum/(a + b) or difference/(b − a)

Find the value of one “part” first.

EXAMPLE Ratio 3 : 5, sum 96 → k = 12 → **36 and 60**.

© 2026 LearnFrenzy · Free for personal study

Sum and product

MOST USED

$$a^2 + b^2 = S^2 - 2P;\ (a - b)^2 = S^2 - 4P;\ 1/a + 1/b = S/P$$

You rarely need the numbers themselves – these identities go straight to the answer.

S – a + b **P** – a × b

EXAMPLE Sum 15, product 56 → $a^2 + b^2 = 225 - 112 =$ **113** (the numbers are 7 and 8).

Difference of squares

$$a^2 - b^2 = (a + b)(a - b)$$

Given the difference of the numbers and of their squares, divide to get the sum.

EXAMPLE Numbers differ by 5, squares differ by 235 → sum = 47 → **26 and 21**.

Consecutive numbers

Sums of consecutive numbers

MOST USED

Sum of k consecutive (or consecutive odd / even) numbers = k × middle number

Divide the sum by the count to get the middle, then step out by 1 or 2.

EXAMPLE Three consecutive even numbers add to 138 → middle 46 → **44, 46, 48**.

Products of consecutive numbers

$$n(n + 1) = P \rightarrow n \approx \sqrt{P} \text{ (round down)}$$

The square root of the product lies between the two numbers.

EXAMPLE Product of two consecutive numbers is 552 → $\sqrt{552} \approx 23.5$ → **23 and 24**.

Digit problems

Reversing a two-digit number

MOST USED

$$(10a + b) - (10b + a) = 9(a - b);\ (10a + b) + (10b + a) = 11(a + b)$$

A reversal difference is always a multiple of 9; a reversal sum, of 11.

EXAMPLE If the digits differ by 3, number and reverse differ by 9 × 3 = **27** (e.g. 74 – 47).

© 2026 LearnFrenzy · Free for personal study

Reversing a three-digit number

> **(100a + 10b + c) − (100c + 10b + a) = 99(a − c)**

The middle digit cancels out.

EXAMPLE Hundreds digit exceeds units digit by 4 → difference = 99 × 4 = **396** (e.g. 844 − 448).

Number from conditions on its digits

MOST USED

> **Write N = 10a + b, turn each condition into an equation in a and b, solve with 1 ≤ a ≤ 9, 0 ≤ b ≤ 9**

Digits are whole numbers 0–9, which often leaves only one possibility.

EXAMPLE A number is 4 times its digit sum, and adding 27 reverses it → b = 2a and b − a = 3 → **36**.

Fractions, parts and mistakes

Wrong operation

MOST USED

> **Wrong result − right result = N × (wrong factor − right factor)**

Dividing by a/b is the same as multiplying by b/a.

EXAMPLE Divided by 3/5 instead of multiplied by 3/5, the answer was 64 too big → N × (5/3 − 3/5) = 64 → N = **60**.

Splitting a number into parts

> **If (1/p) of one part = (1/q) of the other, the parts are in the ratio p : q**

Equal fractions of two parts make the parts proportional to the denominators.

EXAMPLE Divide 80 so that 1/3 of one part equals 1/5 of the other → 3 : 5 → **30 and 50**.

Heads, legs, marks and coins

Heads and legs

MOST USED

> **Four-legged animals = (legs − 2 × heads)/2**

Pretend all are two-legged; the extra legs come in pairs from the four-legged ones.

EXAMPLE 48 heads, 140 legs (hens and cows) → cows = (140 − 96)/2 = **22**, hens 26.

© 2026 LearnFrenzy · Free for personal study

Negative marking

> **Score = (correct × marks per right) − (wrong × marks lost per wrong)**

With all questions attempted, wrong = total − correct.

EXAMPLE 60 questions, +4 / −1, score 130 → 4c − (60 − c) = 130 → c = **38**.

Coins in a ratio

> **Value of one "set" of coins in the ratio → number of sets = total ÷ value of one set**

Convert paise to rupees before adding.

EXAMPLE ₹1, 50p and 25p coins in ratio 5 : 6 : 8 total ₹210 → one set = 5 + 3 + 2 = ₹10 → 21 sets → **105** one-rupee coins.

Numbers in AP and GP

Three numbers in AP

MOST USED

> **Take a − d, a, a + d (sum = 3a)**

The sum gives the middle number at once.

EXAMPLE Sum 27, product 504 → a = 9, $9(81 - d^2) = 504$ → d = 5 → **4, 9, 14**.

Three numbers in GP

> **Take a/r, a, ar (product = a^3)**

The product gives the middle number at once.

EXAMPLE Product 216, sum 19 → a = 6, 6/r + 6 + 6r = 19 → r = 3/2 → **4, 6, 9**.

© 2026 LearnFrenzy · Free for personal study

3 Shortcut tricks & smart techniques

Each trick says when to use it – that matters as much as the trick

Test the options (back-solving)

MOST USED

USE WHEN Digit puzzles and two-condition questions with numeric options.

1. Check the easiest condition first (digit sum, divisibility, reversal).
2. Only the options that pass go on to the second condition.

EXAMPLE "Digit sum 11, number exceeds its reverse by 45": only 83 passes both (8 + 3 = 11, 83 – 38 = **45**).

Work backwards

MOST USED

USE WHEN "Think of a number" chains of operations.

1. Start from the final result.
2. Undo each step in reverse order: ÷ for ×, – for +.

EXAMPLE Doubled, 9 added, result trebled gives 75 → 75 ÷ 3 = 25 → 25 – 9 = 16 → 16 ÷ 2 = **8**.

Ratio and difference

USE WHEN Two numbers in a ratio with a known difference.

1. One part = difference ÷ (difference of ratio terms).
2. Multiply back.

EXAMPLE Ratio 4 : 7, difference 36 → one part = 12 → **48 and 84**.

Product from sum and difference

USE WHEN Sum and difference are given but the product (or squares) is asked.

1. $(a + b)^2 - (a - b)^2 = 4ab$.
2. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$.

EXAMPLE Sum 20, difference 4 → product = (400 – 16)/4 = **96** (12 × 8).

Reversal arithmetic

USE WHEN A number and its reverse are compared.

1. Difference ÷ 9 = difference of the digits.
2. Sum ÷ 11 = sum of the digits.
3. Combine with any other digit fact.

EXAMPLE Number + reverse = 121 → digit sum 11; difference 27 → digits differ by 3 → **74 or 47**.

© 2026 LearnFrenzy · Free for personal study

Assume everyone is the same (heads and legs)

USE WHEN Two kinds of items with a total count and a total of a second feature.

1. Assume all items are the cheaper / fewer-legged kind.
2. The shortfall divided by the difference per item gives the other kind.

EXAMPLE 35 heads, 94 legs (rabbits and chickens): all chickens → 70 legs; 24 extra ÷ 2 = **12 rabbits**.

Negative marking: count the loss

USE WHEN All questions attempted, score known.

1. Maximum score − actual score = total loss.
2. Each wrong answer loses (marks for right + penalty).
3. Wrong answers = loss ÷ that amount.

EXAMPLE 50 questions, +3 / −1, score 102: loss = 150 − 102 = 48; each wrong costs 4 → 12 wrong, **38 correct**.

© 2026 LearnFrenzy · Free for personal study

4 Solved examples

The full method on the left, the exam shortcut on the right

1 TWO NUMBERS

The sum of two numbers is 45 and their difference is 1/9 of their sum. Find their product.

STEP BY STEP

1. Difference = 45/9 = 5.
2. Numbers = (45 + 5)/2 = 25 and (45 − 5)/2 = 20.
3. Product = 500.

SHORTCUT

$4ab = S^2 - D^2 = 2{,}025 - 25 = 2{,}000 \rightarrow ab = 500$.

Why it works: The product follows from the sum and difference without finding the numbers.

Answer: 500

2 SQUARES

Two numbers add to 17 and multiply to 72. Find the sum of their squares.

STEP BY STEP

1. The numbers are 9 and 8 (9 + 8 = 17, 9 × 8 = 72).
2. 81 + 64 = 145.

SHORTCUT

$a^2 + b^2 = S^2 - 2P = 289 - 144 = 145$.

Why it works: $(a + b)^2 = a^2 + b^2 + 2ab$, so subtract 2ab.

Answer: 145

3 CONSECUTIVE

The product of two consecutive odd numbers is 483. Find their sum.

STEP BY STEP

1. Let them be n and n + 2: n(n + 2) = 483.
2. $n^2 + 2n - 483 = 0 \rightarrow (n - 21)(n + 23) = 0$.
3. n = 21 → numbers 21 and 23 → sum 44.

SHORTCUT

$\sqrt{483} \approx 22$ → the odd numbers either side are 21 and 23 → 44.

Why it works: The square root of the product is the number between the two.

Answer: 44

4 DIGITS

The sum of the digits of a two-digit number is 9. When the digits are reversed, the number increases by 63. Find the number.

STEP BY STEP

1. Let the number be 10a + b: a + b = 9.
2. Reversed − original = 9(b − a) = 63 → b − a = 7.
3. a = 1, b = 8 → 18.

SHORTCUT

63 ÷ 9 = 7 → digits 1 and 8, smaller first because the number increased → 18.

Why it works: Reversing changes a two-digit number by 9 × (difference of digits).

Answer: 18

© 2026 LearnFrenzy · Free for personal study

5 THREE DIGITS

In a three-digit number the hundreds digit is twice the units digit, and the tens digit equals the difference of the other two. Subtracting the reversed number gives 396. Find the number.

STEP BY STEP

1. Number − reverse = 99(a − c) = 396 → a − c = 4.
2. a = 2c → 2c − c = 4 → c = 4, a = 8.
3. b = a − c = 4 → 844.

SHORTCUT

396 ÷ 99 = 4; a = 2c and a − c = 4 → (8, 4) → 844.

Why it works: The middle digit disappears from a three-digit reversal difference.

Answer: 844

6 WRONG OPERATION

A number was to be multiplied by 4/7, but by mistake it was divided by 4/7. The answer was 66 more than the correct one. Find the number.

STEP BY STEP

1. Wrong result = N × 7/4; right result = N × 4/7.
2. N(7/4 − 4/7) = N × 33/28 = 66.
3. N = 66 × 28/33 = 56.

SHORTCUT

$(a/b - b/a) = (a^2 - b^2)/ab = 33/28 \rightarrow N = 66 \div 33/28 = 56$.

Why it works: Dividing by 4/7 is multiplying by 7/4; the gap is N times the difference of the two factors.

Answer: 56

7 FRACTIONS

3/5 of a number exceeds 2/7 of the same number by 33. Find the number.

STEP BY STEP

1. (3/5 − 2/7)N = 33.
2. (21 − 10)/35 = 11/35 → (11/35)N = 33.
3. N = 33 × 35/11 = 105.

SHORTCUT

Difference of fractions = 11/35 → N = 33 × 35/11 = 105.

Why it works: Both fractions are of the same number, so subtract the fractions first.

Answer: 105

8 COINS

A bag has ₹1, 50-paise and 25-paise coins in the ratio 2 : 3 : 4. The total value is ₹180. How many 50-paise coins are there?

STEP BY STEP

1. Let the counts be 2k, 3k, 4k.
2. Value = 2k + 1.5k + k = 4.5k = 180 → k = 40.
3. 50-paise coins = 3 × 40 = 120.

SHORTCUT

One set (2 + 3 + 4 coins) is worth ₹4.50 → 180/4.5 = 40 sets → 120.

Why it works: A "set" in the ratio has a fixed value; the number of sets scales everything.

Answer: 120

© 2026 LearnFrenzy · Free for personal study

9 NEGATIVE MARKING

A test has 75 questions with +2 for a correct answer and −0.5 for a wrong one. A student attempts all and scores 110. How many were correct?

STEP BY STEP

1. $2c - 0.5(75 - c) = 110$.
2. $2.5c - 37.5 = 110 \rightarrow c = 59$.

SHORTCUT

Loss = 150 − 110 = 40; each wrong costs 2.5 → 16 wrong → 59 correct.

Why it works: A wrong answer loses the 2 marks it could have earned plus the 0.5 penalty.

Answer: 59

10 AP TRIPLE

Three numbers in AP add to 24 and the sum of their squares is 210. Find the numbers.

STEP BY STEP

1. Take $a - d$, a, $a + d$: $3a = 24 \rightarrow a = 8$.
2. $(8 - d)^2 + 64 + (8 + d)^2 = 192 + 2d^2 = 210$.
3. $d^2 = 9 \rightarrow d = 3 \rightarrow 5, 8, 11$.

SHORTCUT

Middle = 24/3 = 8; extra squares $210 - 3 \times 64 = 18 = 2d^2 \rightarrow d = 3$.

Why it works: Symmetric terms make the d terms cancel in the sum and pair up in the squares.

Answer: 5, 8, 11

11 GP TRIPLE

Three numbers in GP have a product of 1,000 and a sum of 35. Find them.

STEP BY STEP

1. Take a/r, a, ar: $a^3 = 1{,}000 \rightarrow a = 10$.
2. $10/r + 10 + 10r = 35 \rightarrow 10r^2 - 25r + 10 = 0$.
3. $2r^2 - 5r + 2 = 0 \rightarrow r = 2$ (or 1/2) → 5, 10, 20.

SHORTCUT

Middle = $\sqrt[3]{1{,}000} = 10$; the other two multiply to 100 and add to 25 → 5 and 20.

Why it works: In a GP the outer terms multiply to the square of the middle term.

Answer: 5, 10, 20

© 2026 LearnFrenzy · Free for personal study

5 Important exam tips

Patterns, traps and habits that save marks

Frequently asked question patterns

- Two numbers from a sum with a difference, ratio or product.
- Consecutive (odd / even) numbers with a given sum or product.
- Digit puzzles with reversal of two- or three-digit numbers.
- Fractions or percentages of a number compared with each other.
- “Wrong operation” mistakes – divided instead of multiplied.
- Heads and legs, coins in a ratio, negative marking.
- Three numbers in AP or GP.

Common mistakes to avoid

- Writing “7 less than x” as 7 – x.
- Treating a two-digit number with digits a, b as a + b instead of 10a + b.
- Forgetting that a reversed number’s leading digit cannot be 0.
- Mixing paise and rupees in coin questions.
- Ignoring the “all questions attempted” condition in negative-marking questions.
- Taking the wrong root of a quadratic (negative or non-integer values).

Points to remember

- Reversal difference = 9 × (digit difference); reversal sum = 11 × (digit sum).
- Three-digit reversal difference = 99 × (first – last digit).
- Sum of k consecutive numbers = k × middle.
- Dividing by a/b = multiplying by b/a.
- In a GP of three, middle^2 = product of the outer two.

Speed and accuracy

- Test options against the easiest condition first.
- Use identities ($S^2 - 2P$, $S^2 - 4P$) instead of finding the numbers.
- Work backwards for operation chains.
- Use the “assume all” method for heads-and-legs and marking questions.

Preparation strategy

- Practise translating sentences into equations before solving them.
- Practise on LearnFrenzy in order: Easy → Medium → Hard → Exam Pattern Mix → Data Sufficiency.
- Aim for 45 seconds per question; option testing often takes less.
- Keep an error log of misread phrases (“less than”, “of”, “exceeds”).
- Read the Quick Revision Sheet (next page) the night before the exam.

© 2026 LearnFrenzy · Free for personal study

6 Quick revision sheet

Everything on one page for the night before the exam

Formulas

Larger / smaller	**(S + D)/2 / (S − D)/2**	Ratio a : b, sum S	**ak, bk; k = S/(a + b)**
$a^2 + b^2$	**$S^2 - 2P$**	$(a - b)^2$	**$S^2 - 4P$**
1/a + 1/b	**S/P**	4ab	**$S^2 - D^2$**
k consecutive	**sum = k × middle**	Two-digit	**10a + b**
Reverse − number	**9(a − b)**	Number + reverse	**11(a + b)**
Three-digit reversal	**99(a − c)**	Wrong operation	**gap = N × (factor difference)**
Four-legged	**(legs − 2 × heads)/2**	Wrong answers	**loss/(right + penalty)**
AP triple	**a − d, a, a + d**	GP triple	**a/r, a, ar**

Shortcut tricks

- Easiest condition first on options.
- Undo steps in reverse order.
- Difference ÷ 9 → digit gap.
- Sum ÷ 11 → digit sum.
- Assume all are one kind.
- Middle of AP = sum/3.

Must-know numbers

Reversal gaps	9 × 1…8 = 9 · 18 · 27 · 36 · 45 · 54 · 63 · 72
Reversal sums	11 × 2…18 = 22 · 33 · 44 · 55 · 66 · 77 · 88 · 99 · 110 · 121 …
Consecutive products	20·21 = 420 · 21·22 = 462 · 22·23 = 506 · 23·24 = 552 · 24·25 = 600 · 25·26 = 650
Odd products	19·21 = 399 · 21·23 = 483 · 23·25 = 575 · 25·27 = 675

© 2026 LearnFrenzy · Free for personal study

7 A message from LearnFrenzy

For everyone preparing for placements, competitive and government exams

Keep going – you are closer than you think

Success in examinations is not just about how many hours you study. It is about how effectively you understand concepts, practise questions, and improve yourself every day.

Every formula you learn, every problem you solve, and every mistake you correct brings you one step closer to your goal.

Whether you are preparing for campus placements, competitive examinations, or government jobs, remember that consistency, confidence, and smart preparation make a real difference.

Don’t just memorize formulas. Understand the concepts, practise smart techniques, and develop the confidence to solve problems independently.

Your hard work today can create the opportunities you dream of tomorrow.

— Team LearnFrenzy

Practise **Problems on Numbers** free on LearnFrenzy – questions with instant explanations →

© 2026 LearnFrenzy · Free for personal study