LF LearnFrenzy

QUANTITATIVE APTITUDE · FREE STUDY MATERIAL

HCF and LCM

Formulas & Shortcut Tricks

Every important formula, shortcut and exam trick for HCF and LCM in one place – with solved examples, common mistakes and a one-page revision sheet.

WHAT'S INSIDE

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USEFUL FOR

- Campus placements (TCS, Infosys, Wipro, Accenture)
- SSC CGL / CHSL / MTS
- Bank PO & Clerk (IBPS, SBI, RBI)
- Railways (RRB NTPC, Group D)
- UPSC CSAT & state PSC exams

Practise HCF and LCM free on LearnFrenzy – questions with instant explanations →

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1 Topic introduction

What it is, why it matters and how it is asked

The HCF (Highest Common Factor, also called GCD) of numbers is the largest number that divides all of them exactly. The LCM (Lowest Common Multiple) is the smallest number that all of them divide exactly. One looks for the biggest common piece; the other for the first common meeting point.

That is why HCF answers “largest size” questions – tiles, ropes, groups, containers – and LCM answers “when again” questions – bells, traffic lights, runners on a track, buses leaving together.

Why it matters in exams

- SSC and Railway papers ask 1–3 questions every time, usually remainder or bell-type problems.
- Placement tests (TCS NQT, Infosys, AMCAT) use HCF/LCM pairs, ratio problems and track-meeting puzzles.
- The ideas reappear in fractions, time and work, and simplification, so this chapter pays off twice.

Where it is used

- Scheduling: when will two events with different cycles happen together again?
- Packing and cutting: the largest equal size that wastes nothing.
- Adding fractions needs the LCM of denominators; simplifying needs the HCF.

Common question patterns

Basic HCF / LCM
By prime factorisation or Euclid’s division method.

HCF × LCM
Find the second number, or check whether given values are possible.

Same remainder
Greatest number dividing several numbers with equal remainders.

Constant shortfall
Smallest number leaving remainders each k less than the divisors.

Bells and lights
When do they ring or flash together again? How many times in an hour?

Circular track
When do runners meet again at the starting point?

Tiles and groups
Largest square tile, fewest rows, largest container.

Pairs
How many pairs have a given HCF with a given sum, product or LCM?

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2 Important formulas

Cards marked MOST USED are the ones to revise first

Key terms

Term	Meaning	Example
Factor	A number that divides exactly	Factors of 12: 1, 2, 3, 4, 6, 12
Multiple	A number obtained by multiplying	Multiples of 12: 12, 24, 36, …
HCF / GCD	Largest common factor	HCF(12, 18) = 6
LCM	Smallest common multiple	LCM(12, 18) = 36
Co-prime	HCF is 1; then LCM = product	HCF(8, 15) = 1, LCM = 120

HCF ≤ every number ≤ LCM. The HCF always divides the LCM. Any two consecutive numbers are co-prime.

Finding HCF and LCM

Prime factorisation method

MOST USED

HCF = product of common primes with the lowest powers; LCM = product of all primes with the highest powers

Write every number as powers of primes, then compare the powers prime by prime.

EXAMPLE 360 = $2^3 \times 3^2 \times 5$ and 84 = $2^2 \times 3 \times 7$ → HCF = $2^2 \times 3$ = **12**, LCM = $2^3 \times 3^2 \times 5 \times 7$ = **2,520**.

Euclid's division method

MOST USED

HCF(a, b) = HCF(b, a mod b); repeat until the remainder is 0 – the last divisor is the HCF

Best for large numbers that are hard to factorise.

EXAMPLE HCF(1071, 462): 1071 = 2 × 462 + 147; 462 = 3 × 147 + 21; 147 = 7 × 21 → **21**.

More than two numbers

HCF(a, b, c) = HCF(HCF(a, b), c); LCM(a, b, c) = LCM(LCM(a, b), c)

Combine two at a time.

EXAMPLE HCF(84, 126, 210) = HCF(42, 210) = **42**; LCM(12, 18, 30) = LCM(36, 30) = **180**.

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The HCF–LCM relation

Product rule (two numbers only)

MOST USED

> **HCF × LCM = first number × second number**

Know any three of the four values and the fourth follows. It does **not** hold for three or more numbers.

EXAMPLE HCF 12, LCM 2,520, one number 360 → other = 12 × 2,520 ⁄ 360 = **84**.

Numbers with a known HCF

MOST USED

> **If HCF = H, the numbers are H·x and H·y with x and y co-prime; LCM = H·x·y**

Divide out the HCF and you are left with co-prime parts – this turns most “pairs” questions into counting.

H – the HCF **x, y** – co-prime whole numbers

EXAMPLE HCF 16, sum 128 → x + y = 8 with x, y co-prime: (1, 7), (3, 5) → **2** pairs.

Fractions and decimals

HCF and LCM of fractions

MOST USED

> **HCF = HCF of numerators ⁄ LCM of denominators; LCM = LCM of numerators ⁄ HCF of denominators**

Write each fraction in lowest terms first.

EXAMPLE 2/3, 8/9, 16/81: HCF = 2 ⁄ 81, LCM = 16 ⁄ 3 = **5 1/3**.

HCF and LCM of decimals

> **Make the decimal places equal, find the HCF/LCM of the whole numbers, then put the decimal point back**

Same number of places in every value before you start.

EXAMPLE 0.54, 1.8, 7.2 → 54, 180, 720 → HCF 18 → **0.18**.

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Remainder problems

Greatest number leaving a given remainder

MOST USED

Same remainder r: HCF(a − r, b − r, …). Different remainders r_1, r_2: HCF(a − r_1, b − r_2)

Remove the remainders and the divisor must divide what is left exactly. It must also be larger than every remainder.

EXAMPLE Divides 2,025 and 1,530 leaving 5 and 10 → HCF(2,020, 1,520) = **20**.

Greatest number leaving the same (unknown) remainder

MOST USED

HCF of the differences: HCF(b − a, c − b, c − a)

Equal remainders cancel when you subtract, so the divisor divides every difference.

EXAMPLE 1,232, 1,880, 2,312: differences 648, 432, 1,080 → HCF = **216** (remainder 152 each time).

Smallest number leaving the same remainder r

LCM(a, b, c) + r

The LCM is exactly divisible by all; adding r gives remainder r for each.

EXAMPLE Divided by 12, 15, 20 leaves 7 each time → 60 + 7 = **67**.

Constant shortfall

MOST USED

Remainders are each k less than the divisors → number = LCM − k

The number is k short of a common multiple.

EXAMPLE Divided by 6, 8, 10 leaves 4, 6, 8 (each 2 short) → 120 − 2 = **118**.

Greatest or smallest n-digit multiple

Greatest: largest n-digit number − (its remainder on the LCM). Smallest: smallest n-digit + (LCM − remainder)

Work with the LCM as a single divisor.

EXAMPLE Greatest 4-digit number divisible by 12, 18, 27: LCM 108, 9,999 leaves 63 → **9,936**.

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Meeting again, tiles and number results

Bells, lights and runners

MOST USED

Together again after LCM of the intervals (or lap times)

Bring all intervals to the same unit first – seconds with seconds.

EXAMPLE Bells every 12, 18 and 30 minutes ring together at 9:00 → again after 180 min, at **12:00**.

Largest square tile

Side = HCF of length and breadth; number of tiles = (L/side) × (B/side)

Convert to the smallest unit (cm) so both sides are whole numbers.

EXAMPLE 6.24 m × 4.68 m → HCF(624, 468) = 156 cm → 4 × 3 = **12** tiles.

HCF of numbers of the form $2^n - 1$

$\text{HCF}(2^m - 1, 2^n - 1) = 2^{\text{HCF}(m, n)} - 1$ (works for any base $a > 1$)

A favourite of placement tests – no division needed.

EXAMPLE $\text{HCF}(2^{12} - 1, 2^{18} - 1) = 2^6 - 1 =$ **63**.

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3 Shortcut tricks & smart techniques

Each trick says when to use it – that matters as much as the trick

Which formula? – the remainder table

MOST USED

USE WHEN Any question that mentions remainders – decide the case before you calculate.

The question says	Answer
Greatest number, remainder r in each case	HCF(a – r, b – r, …)
Greatest number, same remainder (not given)	HCF of the differences
Smallest number, remainder r in each case	LCM + r
Smallest number, remainders each k less than divisors	LCM – k
Smallest number, remainder r, and also divisible by d	LCM·m + r, with m the smallest value making it divisible by d

EXAMPLE Leaves 3 when divided by 5, 6, 7, 8 but is divisible by 9: 840m + 3 ≡ 3m + 3 (mod 9) → m = 2 → **1,683**.

Kill impossible options

MOST USED

USE WHEN Questions that give an HCF and an LCM, or ask which pair is possible.

1. The LCM must be a multiple of the HCF.
2. Each number must be a multiple of the HCF and a factor of the LCM.
3. For two numbers, HCF × LCM must equal their product.

EXAMPLE HCF 12 and LCM 400 cannot both be true, because 400 ÷ 12 is **not a whole number**.

Subtraction shortcut for large HCFs

USE WHEN Two close, large numbers.

1. HCF(a, b) = HCF(a – b, b), so the HCF divides the difference.
2. Factorise the small difference and test its factors on one number.

EXAMPLE HCF(4,199, 4,301): difference 102 = 2 × 3 × 17; 4,199 = 17 × 247 and 4,301 = 17 × 253 → **17**.

Differences give the candidates

USE WHEN HCF of three or more numbers that are close together.

1. Take the smallest difference between the numbers.
2. The HCF must be one of its factors – test from the largest down.

EXAMPLE HCF(391, 425, 527): differences 34 and 102 → try 34 (fails), 17 (works) → **17**.

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Counting pairs with a given HCF and LCM

USE WHEN "How many pairs of numbers have HCF H and LCM L?"

1. Write the numbers as H·x and H·y.
2. Then x·y = L/H with x, y co-prime.
3. Split L/H into co-prime factor pairs – each distinct prime power must go wholly to one side.

EXAMPLE HCF 12, LCM 720 → x·y = 60 = 4 × 3 × 5 → (1, 60), (3, 20), (4, 15), (5, 12) → **4** pairs.

Ratio with HCF

USE WHEN Numbers are given in a ratio together with their HCF or LCM.

1. Numbers in ratio a : b (a, b co-prime) are h·a and h·b, where h is the HCF.
2. Their LCM is h·a·b.

EXAMPLE Ratio 3 : 4 and LCM 180 → 12h = 180 → h = 15 → numbers 45 and 60, sum **105**.

Common multiples in a range

USE WHEN "How many numbers up to N are divisible by both a and b?"

1. Divisible by both = divisible by their LCM (not their product, unless co-prime).
2. Count = ⌊N/LCM⌋.

EXAMPLE 1 to 1,000 divisible by 12 and 18: LCM 36 → ⌊1000/36⌋ = **27**.

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4 Solved examples

The full method on the left, the exam shortcut on the right

1 FACTORISATION

Find the HCF and LCM of 144, 180 and 192.

STEP BY STEP

1. $144 = 2^4 \times 3^2$; $180 = 2^2 \times 3^2 \times 5$; $192 = 2^6 \times 3$.
2. HCF: lowest powers of common primes = $2^2 \times 3 = 12$.
3. LCM: highest powers of all primes = $2^6 \times 3^2 \times 5 = 2{,}880$.

SHORTCUT

HCF divides the differences 36 and 12 → try 12: it divides all three. LCM = 192 × 15 = 2,880 (192 needs 3 more and 5).

Why it works: The HCF must divide every difference, which narrows the candidates instantly.

Answer: HCF 12, LCM 2,880

2 PAIRS

The product of two numbers is 2,160 and their HCF is 12. How many such pairs exist?

STEP BY STEP

1. Numbers = 12x and 12y with x, y co-prime.
2. $144xy = 2{,}160 \rightarrow xy = 15$.
3. Co-prime splits of 15: (1, 15) and (3, 5).
4. Pairs: (12, 180) and (36, 60).

SHORTCUT

Product ÷ $HCF^2 = 15$; co-prime factor pairs of 15 → 2.

Why it works: After removing the HCF from both numbers, the remaining parts must share no factor.

Answer: 2

3 SAME REMAINDER

Find the greatest number that divides 1,232, 1,880 and 2,312 leaving the same remainder in each case.

STEP BY STEP

1. Let the remainder be r; then the number divides 1,880 − 1,232 = 648, 2,312 − 1,880 = 432 and 2,312 − 1,232 = 1,080.
2. $648 = 2^3 \times 3^4$, $432 = 2^4 \times 3^3$.
3. HCF = $2^3 \times 3^3 = 216$, which also divides 1,080.

SHORTCUT

HCF of two differences is enough: HCF(648, 432) = 216.

Why it works: Subtracting removes the unknown remainder, leaving exact multiples of the divisor.

Answer: 216

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4 REMAINDER + DIVISIBILITY

Find the smallest number that leaves remainder 4 when divided by 12, 16, 18 and 30, but is exactly divisible by 7.

STEP BY STEP

1. LCM(12, 16, 18, 30) = 720, so the number is 720m + 4.
2. 720 leaves 6 on division by 7, so 720m + 4 ≡ 6m + 4 (mod 7).
3. 6m + 4 ≡ 0 → 6m ≡ 3 → m = 4 (6 × 4 = 24 ≡ 3).
4. Number = 720 × 4 + 4 = 2,884.

SHORTCUT

Try m = 1, 2, 3, 4 on 720m + 4: 724, 1,444, 2,164, 2,884 – only 2,884 = 7 × 412.

Why it works: Every number with remainder 4 for all four divisors has the form LCM × m + 4.

Answer: 2,884

5 BELLS

Four bells toll at intervals of 6, 8, 12 and 18 seconds. They toll together now. How many times will they toll together in the next 30 minutes?

STEP BY STEP

1. They toll together every LCM(6, 8, 12, 18) = 72 seconds.
2. 30 minutes = 1,800 seconds.
3. 1,800 ÷ 72 = 25.

SHORTCUT

Time ÷ LCM = 1,800⁄72 = 25 (add 1 only if the starting toll is to be counted).

Why it works: Each common toll is one more LCM-length gap after the start.

Answer: 25

6 CIRCULAR TRACK

A, B and C start together and take 252, 308 and 198 seconds per lap. When will they next meet at the starting point?

STEP BY STEP

1. $252 = 2^2 \times 3^2 \times 7$; $308 = 2^2 \times 7 \times 11$; $198 = 2 \times 3^2 \times 11$.
2. LCM = $2^2 \times 3^2 \times 7 \times 11$ = 2,772 s.
3. 2,772 s = 46 min 12 s.

SHORTCUT

LCM of lap times, then convert seconds to minutes.

Why it works: Each runner is at the start only at multiples of their own lap time.

Answer: 46 min 12 s

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7 FRACTIONS

Three lights flash every 3/4, 9/10 and 15/16 of a minute. They flash together now. After how long will they next flash together?

STEP BY STEP

1. LCM of fractions = LCM(3, 9, 15)/HCF(4, 10, 16).
2. LCM(3, 9, 15) = 45; HCF(4, 10, 16) = 2.
3. LCM = 45/2 = 22 1/2 minutes.

SHORTCUT

"Together again" → LCM; for fractions, LCM of tops over HCF of bottoms.

Why it works: 45/2 is the smallest time that is a whole number of each interval (30, 25 and 24 flashes).

Answer: 22 1/2 minutes

8 TILES

A courtyard is 18 m 72 cm long and 13 m 20 cm wide. Find the least number of identical square tiles that cover it exactly.

STEP BY STEP

1. In cm: 1,872 and 1,320.
2. Largest tile side = HCF(1,872, 1,320) = 24 cm.
3. Tiles = (1,872/24) × (1,320/24) = 78 × 55 = 4,290.

SHORTCUT

Least tiles ↔ largest tile ↔ HCF of the sides.

Why it works: The tile side must divide both length and breadth exactly; the largest such side gives the fewest tiles.

Answer: 4,290

9 RATIO

Two numbers are in the ratio 5 : 7 and their LCM is 1,260. Find their sum.

STEP BY STEP

1. Numbers = 5h and 7h, where h is the HCF.
2. LCM = 35h = 1,260 → h = 36.
3. Numbers 180 and 252; sum = 432.

SHORTCUT

Sum = (5 + 7) × LCM/35 = 12 × 36 = 432.

Why it works: 5 and 7 are co-prime, so the LCM is h × 5 × 7.

Answer: 432

10 N-DIGIT NUMBER

Find the greatest 5-digit number that leaves remainder 7 when divided by 15, 24 and 36.

STEP BY STEP

1. LCM(15, 24, 36) = 360.
2. 99,999 ÷ 360 leaves 279, so the greatest 5-digit multiple of 360 is 99,720.
3. Add the remainder: 99,720 + 7 = 99,727.

SHORTCUT

(Largest multiple of the LCM) + r – check it is still 5 digits.

Why it works: Adding r to a common multiple gives remainder r for every divisor.

Answer: 99,727

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11 NUMBER THEORY

Find the HCF of $2^{24} - 1$ and $2^{36} - 1$.

STEP BY STEP

1. HCF($2^m - 1$, $2^n - 1$) = $2^{HCF(m, n)} - 1$.
2. HCF(24, 36) = 12.
3. Answer = $2^{12} - 1$ = 4,095.

SHORTCUT

HCF of the powers, then 2 to that power minus 1.

Why it works: Euclid's steps on the numbers mirror Euclid's steps on the powers.

Answer: 4,095

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5 Important exam tips

Patterns, traps and habits that save marks

Frequently asked question patterns

- Greatest number with given or equal remainders (SSC favourite).
- Smallest number with a constant remainder or constant shortfall, sometimes also divisible by another number.
- Bells, traffic lights and runners meeting again; how many times in a given period.
- HCF × LCM = product: find the other number, or test given pairs.
- Pairs with a given HCF and sum, product or LCM.
- Largest tile, rope piece or container; fewest groups or rows.
- HCF and LCM of fractions and decimals.

Common mistakes to avoid

- Using HCF × LCM = product for three numbers – it works only for two.
- Mixing units – minutes with seconds, metres with centimetres.
- Forgetting that the divisor must be larger than every remainder.
- Answering LCM for a “largest size” question (that needs HCF), or the reverse.
- For fractions, taking LCM of denominators for the LCM – it is the HCF of denominators.
- Counting the starting moment as a “meeting” when the question says “after”.
- Counting pairs whose parts are not co-prime (e.g. x = 2, y = 4 when the HCF is fixed).

Points to remember

- HCF divides the LCM, and both relate as HCF ≤ number ≤ LCM.
- Co-prime numbers: HCF = 1, LCM = product.
- HCF divides every difference of the numbers.
- LCM(1 to 10) = 2,520; LCM(1 to 6) = 60; LCM(1 to 8) = 840.
- HCF(a, b) = HCF(a – b, b).

Speed and accuracy

- Decide the remainder case first using the table on the tricks page.
- Use differences to guess the HCF, then check one number.
- Kill options that break HCF | LCM before calculating.
- For LCM of several numbers, start with the largest and take its multiples until all divide.

Preparation strategy

- Learn the methods, then the remainder table – most mistakes come from picking the wrong case.
- Practise on LearnFrenzy in order: Easy → Medium → Hard → Exam Pattern Mix → Data Sufficiency.
- Aim for 45 seconds per question; bells and track questions should take under 30.
- Keep an error log noting which case (HCF or LCM, which remainder rule) you confused.
- Read the Quick Revision Sheet (next page) the night before the exam.

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6 Quick revision sheet

Everything on one page for the night before the exam

Formulas

HCF (factorisation)	**common primes, lowest powers**	LCM (factorisation)	**all primes, highest powers**
Euclid	**HCF(a, b) = HCF(b, a mod b)**	Two numbers	**HCF × LCM = a × b**
Known HCF H	**Hx, Hy; x, y co-prime**	LCM with HCF H	**H·x·y**
HCF of fractions	**HCF(num)/LCM(den)**	LCM of fractions	**LCM(num)/HCF(den)**
Greatest, remainder r	**HCF(a − r, b − r)**	Greatest, same rem.	**HCF of differences**
Smallest, remainder r	**LCM + r**	Smallest, shortfall k	**LCM − k**
Meet again	**LCM of intervals**	Largest tile	**HCF of sides**
Ratio a : b, HCF h	**ha, hb; LCM = hab**	$2^m - 1, 2^n - 1$	$2^{HCF(m, n)} - 1$

Shortcut tricks

- "Largest / greatest" → HCF.
- "When again / smallest" → LCM.
- HCF divides every difference.
- LCM must be a multiple of HCF.
- Same units before LCM.
- Pairs: split into co-prime parts.

Must-know numbers

LCM of 1 to n	1-4: 12 · 1-6: 60 · 1-8: 840 · 1-10: 2,520 · 1-12: 27,720
Handy LCMs	(8, 12) 24 · (12, 18) 36 · (12, 15, 20) 60 · (6, 8, 10) 120 · (15, 24, 36) 360
Time	1 min = 60 s · 1 h = 3,600 s · 1 day = 1,440 min
Length	1 m = 100 cm · 1 km = 1,000 m

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7 A message from LearnFrenzy

For everyone preparing for placements, competitive and government exams

Keep going – you are closer than you think

Success in examinations is not just about how many hours you study. It is about how effectively you understand concepts, practise questions, and improve yourself every day.

Every formula you learn, every problem you solve, and every mistake you correct brings you one step closer to your goal.

Whether you are preparing for campus placements, competitive examinations, or government jobs, remember that consistency, confidence, and smart preparation make a real difference.

Don’t just memorize formulas. Understand the concepts, practise smart techniques, and develop the confidence to solve problems independently.

Your hard work today can create the opportunities you dream of tomorrow.

— Team LearnFrenzy

Practise **HCF and LCM** free on LearnFrenzy – questions with instant explanations →

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